ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-04

Today's Target

- ***Binomial Probability Distribution***
- PYQ
- DPP

Probability Distribution

Discrete probability distribution

(Based on Discrete random variable)

(i) Binomial probability distribution 2L

(ii) Poisson probability distribution 2L

Continous probability distribution

(Based on continous radom variable)

(i) Normal probability Distribution 2L

Binomial probability Distribution

Bernoulli's Distribution

In Binomial distribution :

- Number of trials are finite (n)
- All trials are independent
- Each trial has only two outcomes (success or failure)

Binomial probability Distribution

Let a random variable X denote the number of successes in n independent trials in an experiment.

Let p be the probability of success and q be the probablity of failure in a single trial so that

$$p + q = 1$$

r successes can be obtained in n trials in ${}^{n}C_{r}$ ways.

Probability of r successes in n trials

$$P(X = r) = {}^{n}C_{r}\ \underbrace{P(S)\ P(S) \dots\dots\dots P(S)}_{r \text{ times}}\ \underbrace{P(F)\ P(F) \dots\dots\dots P(F)}_{(n-r) \text{ times}}$$

$$P(X = r) = {}^{n}C_{r}\ p\ p\ p \dots\dots\dots p\ \ q\ q\ q \dots\dots\dots q$$

$$\boxed{P(X = r) = {}^{n}C_{r}\ p^{r}\ q^{n-r}}$$

p = Prob. of success
q = Prob. of failure

n = No. of trials
r = No. of success

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Where $p + q = 1$

And $r = 0,\ 1,\ 2,\ 3\ \ldots$

$$P(X = 0) = n_{c_0}\, q^n$$

$$P(X = 1) = n_{c_1}\, p\, q^{n-1}$$

$$P(X = 2) = n_{c_2}\, p^2 q^{n-2}$$

$$\vdots \qquad\qquad \vdots$$

$$P(X = n) = n_{c_n}\, p^n$$

$n_{c_0}\, q^n\ ,\ n_{c_1}\, p\, q^{n-1}\ \ ,\ n_{c_2}\, p^2 q^{n-2}\ \ \ldots\ldots\ldots\ldots\ n_{c_n}\, p^n$ *are successive terms of binomial expansion* $(q + p)^n$

➤ *If n independent trials constitute one experiment and this experiment is repeated N times.*

➤ *The frequuency of success is*

$$N\ n_{c_r}\ p^r\ q^{n-r}$$

And Binomial Distribution is $\boxed{N(q+p)^n}$

Mean, variance and standard deviation of Binomial distribution

(i) Mean of Binomial Distribution,

$$\boxed{\mu = np}$$

First moment about origin

(i) Variance of Binomial Distribution,

$$\boxed{\text{Variance} = npq}$$

Second moment about mean

(i) *Standard Deviation*,

$$\sigma = \sqrt{npq}$$

Recurrence or Recursion formula for Binomial Distribution

$$p(r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q} p(r)$$

Q.1 Comment on the following statement : For a Binomial distribution, mean is 6 and variance is 9.

Mean

$np = 6$ — ①

Variance

$npq = 9$ — ②

Divide ② by ①

$$\frac{\cancel{np}q}{\cancel{np}} = \frac{9}{6}_2^3$$

$$q = \frac{3}{2}$$

$q = 1.5$ (impossible)

Since, $0 \leq q \leq 1$

Given statement is False

Q. 2 *Ten coins are tossed simultaneously. Find the probability of getting at least seven heads.*

$P(H) = \frac{1}{2}$

$p = \frac{1}{2}$

$P(T) = \frac{1}{2}$

$q = \frac{1}{2}$

$n = 10$

$P(X=r) = {}^{n}C_{r} p^{r} q^{n-r}$

$P(r \geq 7) = P(r=7) + P(r=8) + P(r=9) + P(r=10)$

$= {}^{10}C_{7}\left(\frac{1}{2}\right)^{7}\left(\frac{1}{2}\right)^{3} + {}^{10}C_{8}\left(\frac{1}{2}\right)^{8}\left(\frac{1}{2}\right)^{2} + {}^{10}C_{9}\left(\frac{1}{2}\right)^{9}\left(\frac{1}{2}\right)^{1} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$

$= \left(\frac{1}{2}\right)^{10}\left[{}^{10}C_{7} + {}^{10}C_{8} + {}^{10}C_{9} + {}^{10}C_{10}\right]$

$= \left(\frac{1}{2}\right)^{10}\left[\frac{10 \times 9 \times 8}{3 \times 2} + \frac{10 \times 9}{2} + 10 + 1\right]$

$= \left(\frac{1}{2}\right)^{10} \times \left[120 + 45 + 10 + 1\right]$

$= 176 \times \left(\frac{1}{2}\right)^{10}$

$P(r \geq 7) = \frac{11}{64}$

Q.3 *A die is thrown five times. If getting an odd number is a success, find the probability of getting at least four successes.*

$n = 5$

Getting an odd number is a success

$S = \{1, 2, 3, 4, 5, 6\}$

$P = \frac{3}{6} = \frac{1}{2}$

$P = \frac{1}{2}$ $\quad$ $q = \frac{1}{2}$

$P(r \geq 4) = P(r = 4) + P(r = 5)$

$= {}^5C_4 \left(\frac{1}{2}\right)^4 \times \frac{1}{2} + {}^5C_5 \left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^5 \left[{}^5C_4 + {}^5C_5\right]$

$= \frac{1}{32}(5 + 1)$

$= \frac{1}{32} \times \frac{6}{16}^{3}$

$P(r \geq 4) = \frac{3}{16}$

Q.4 *A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.*

$S = \{1, 2, 3, 4, 5, 6\}$

Prob. of success

$$P = \frac{2}{6} \quad 3$$

$$\boxed{P = \frac{1}{3}}$$

Prob. of failure

$$\boxed{q = \frac{2}{3}}$$

$n = 3$

Mean

$$\mu = np = 3 \times \frac{1}{3}$$

$$\boxed{\mu = 1}$$

Variance

$$\text{Var}(x) = npq = 3 \times \frac{1}{3} \times \frac{2}{3}$$

$$\boxed{\text{Var}(x) = \frac{2}{3}}$$

Q.5 *A binomial variable X satisfies the relation* $9P(X = 4) = P(X = 2)$ *when* $n = 6$.

Find the value of the parameter p and $P(X = 1)$.

$n = 6$

We know that

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$P(X=4) = {}^{6}C_{4}\, p^{4} q^{2}$$

$$P(X=2) = {}^{6}C_{2}\, p^{2} q^{4}$$

According to question

$$9P(X=4) = P(X=2)$$

$$9 \times {}^{6}C_{4} \times p^{4} q^{2} = {}^{6}C_{2} \times p^{2} q^{4}$$

$$9 \times \frac{6!}{2! \times 4!} \times p^{2} = \frac{6!}{4! \times 2!} \times q^{2}$$

$$9p^{2} = q^{2}$$

$$9p^{2} = (1-p)^{2}$$

$$9p^2 = (1-p)^2$$

$$9p^2 = 1 + p^2 - 2p$$

$$8p^2 + 2p - 1 = 0$$

$$8p^2 + 4p - 2p - 1 = 0$$

$$4p(2p+1) - 1(2p+1) = 0$$

$$(4p-1)(2p+1) = 0$$

$$\boxed{p = \frac{1}{4}}$$

$$p = -\frac{1}{2} \text{ (Reject)}$$

$$q = 1 - p$$

$$q = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\boxed{q = \frac{3}{4}}$$

$$P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$P(X = 1) = {}^{6}C_{1} \left(\frac{1}{4}\right)^{1} \times \left(\frac{3}{4}\right)^{5}$$

$$P(X = 1) = \frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}} \times \frac{1}{\cancel{4}} \times \frac{3^5}{4^5}$$

$$P(X = 1) = \frac{729}{2048}$$

$$\boxed{P(X = 1) = 0.3559}$$

Q.6 ***Fit a binomial distribution to the following frequency data :***

x:	0	1	2	3	4
f:	24	41	28	5	2

x	f	fx
0	24	0
1	41	41
2	28	56
3	5	15
4	2	8
	$\Sigma f = 100$	$\Sigma fx = 120$

$\Sigma f = N$

Mean

$$\mu = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{120}{100}$$

$$\mu = 1.2$$

For Binomial Distribution

$$\mu = np$$

$$1.2 = 4 \times p$$

$$p = 0.3$$

$$q = 0.7$$

Binomial Distribution

$$N(q+p)^n = 100(0.7+0.3)^4$$

Topic : *Binomial probability distribution*

Q.1 If the mean of a binomial distribution is 3 and the variabce is $\frac{3}{2}$, find the probability of obtaining at least 4 successes.

Q.2 The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$.

If six bombs are dropped, find the probability that (i)exactly two will strike the target, (ii)at least two will trike the target.

Q.3 Fit a binimial distribution to the following frequency data:

x:	0	1	2	3	4
f:	30	62	46	10	2

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