Engineering Mathematics-IV

Module -II: Applications of Partial Differential Equations and Fourier Transform

Chapter-1 :- Applications of Partial Differential Equations

- Method of separation of variables
- Solution of one dimensional heat equation, wave equation
- Two dimensional heat equation (only Laplace Equation) and their application

Chapter-2 :- Fourier Transform

- Complex Fourier transform, Fourier sine transform, Fourier cosine transform, Inverse transform, convolution theorem
- Application of Fourier Transformto solve partial differential equation.

Engineering Mathematics

UNIT : Applications of Partial Differential Equations and Fourier Transform

Today's Target

- Method of separation of variables(When boundary conditions are not given)
- PYQs
- DPP

Calculation of Complementary Function (C.F)

CASE-1 : When A.E has Real and distinct roots

Nature of roots	C.F
(i) One real root m_1	$CF = c_1 e^{m_1 x}$
(ii) Two real and distinct roots m_1, m_2	$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
(iii) Three real and distinct roots m_1, m_2, m_3	$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$

CASE-2 : When A.E has Real and equal roots

Nature of roots	C.F
(i) Two real and equal roots m, m	$CF = (c_1 + c_2 x)e^{mx}$
(ii) Three real and equal roots m, m, m	$CF = (c_1 + c_2 x + c_3 x^2)e^{mx}$

CASE-3 : When A.E has *complex* roots

Nature of roots	C.F
(i) One pair of complex roots $\alpha \pm i\beta$	$CF = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$
(ii) Two pair of complex and equal roots $\alpha \pm i\beta, \alpha \pm i\beta$	$CF = e^{\alpha x}[(c_1 + c_2 x)\cos \beta x + (c_3 + c_4 x)\sin \beta x]$

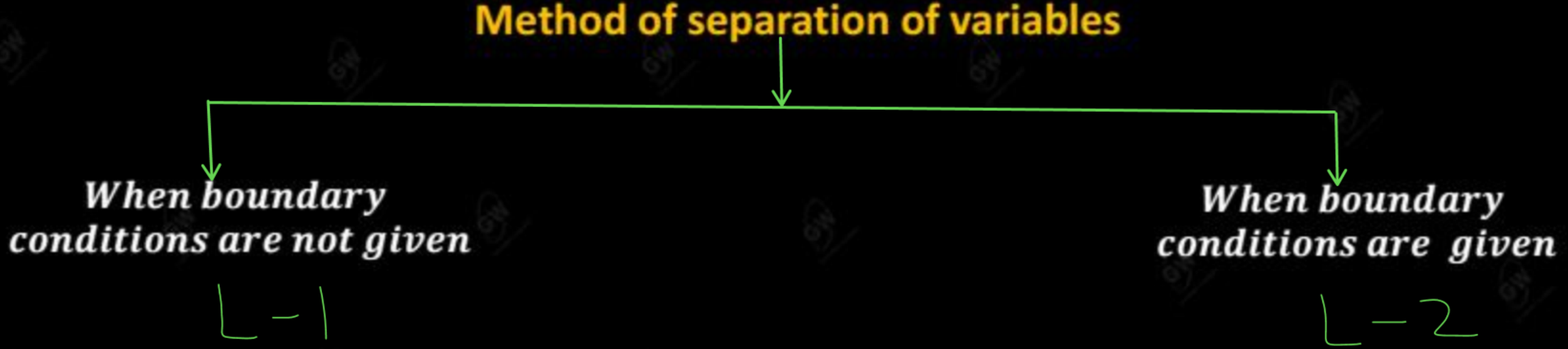
CASE-4 : When A.E has *Irrational* roots

Nature of roots	C.F
(i) One pair of irrational roots $\alpha \pm \sqrt{\beta}$	$CF = c_1 e^{(\alpha+\beta)x} + c_2 e^{(\alpha-\beta)x}$
(ii) Two pair of irrational and equal roots $\alpha \pm \sqrt{\beta}, \quad \alpha \pm \sqrt{\beta}$	$CF = (c_1 + c_2 x)e^{(\alpha+\beta)x} + (c_3 + c_4 x)e^{(\alpha-\beta)x}$

Method of separation of variables

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- *In this method, we assume the solution of the given PDE as the product of the two functions.*
- *Each function is the function of one variable only.*
- *So two ordinary differential equations are formed.*



When boundary conditions are not given

Q.1. Use the method of separation of variables to solve the equation $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$. (GBTU-2012)

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad \text{—— (1)}$$

Let $u = X(x)\,Y(y)$

$$u = XY \quad \text{—— (2)}$$

$$\frac{\partial u}{\partial x} = Y\frac{dX}{dx} = YX'$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = YX''$$

$$\frac{\partial u}{\partial y} = X\frac{dY}{dy} = XY'$$

Put these values in (1)

$$YX'' - 2YX' + XY' = 0 \quad \text{— (3)}$$

Divide by XY

$$\frac{YX''}{XY} - \frac{2YX'}{XY} + \frac{XY'}{XY} = \frac{0}{XY}$$

$$\frac{X''}{X} - 2\frac{X'}{X} + \frac{Y'}{Y} = 0$$

$$\frac{X''}{X} - 2\frac{X'}{X} = -\frac{Y'}{Y}$$

$$\frac{X'' - 2X'}{X} = -\frac{Y'}{Y} = -b^2 \text{ (say)}$$

(1)

$$\frac{X'' - 2X'}{X} = -b^2$$

$$X'' - 2X' = -b^2 X$$

$$X'' - 2X' + b^2 X = 0$$

$$D^2X - 2DX + b^2 X = 0$$

$$(D^2 - 2D + b^2)X = 0$$

Aux. Eqn

$$D^2 - 2D + b^2 = 0$$

$$D^2 - 2D + p^2 = 0$$

$$D = \frac{2 \pm \sqrt{(-2)^2 - 4 \times p^2}}{2 \times 1}$$

$$D = \frac{2 \pm \sqrt{4 - 4p^2}}{2}$$

$$D = \frac{2 \pm 2\sqrt{1-p^2}}{2}$$

$$D = 1 \pm \sqrt{1-p^2}$$

$$CF = c_1 e^{(1+\sqrt{1-p^2})x} + c_2 e^{(1-\sqrt{1-p^2})x}$$

$$PI = 0$$

$$X = CF + PI$$

$$\boxed{X = c_1 e^{(1+\sqrt{1-p^2})x} + c_2 e^{(1-\sqrt{1-p^2})x}}$$

From (4)

$$\frac{-Y'}{Y} = -p^2$$

$$+Y' = +p^2 Y$$

$$Y' = b^2 Y$$

$$Y' - b^2 Y = 0$$

$$DY - b^2 Y = 0$$

$$(D - b^2) Y = 0$$

Aux. Eqn

$$D - b^2 = 0$$

$$D = b^2$$

$$CF = C_3 e^{b^2 y}$$

$$PI = 0$$

$$Y = CF + PI$$

$$\boxed{Y = C_3 e^{b^2 y}}$$

Put X and Y in (2)

$$\boxed{U = \left[C_1 e^{(1+\sqrt{1-b^2})x} + C_2 e^{(1-\sqrt{1-b^2})x} \right] C_3 e^{b^2 y}}$$

Q.2. Use the method of separation of variables to solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$.

(UPTU-2014)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u \quad \text{——} (1)$$

Let $u = X(x)\,Y(y)$

$$u = XY \quad \text{——} (2)$$

$$\frac{\partial u}{\partial x} = Y\frac{dX}{dx} = YX'$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = YX''$$

$$\frac{\partial u}{\partial y} = X\frac{dY}{dy} = XY'$$

Put these values in (1)

$$YX'' = XY' + 2XY$$

Divide by XY

$$\frac{YX''}{XY} = \frac{XY'}{XY} + \frac{2XY}{XY}$$

$$\frac{X''}{X} = \frac{Y'}{Y} + 2$$

$$\frac{X''}{X} = \frac{Y'}{Y} + 2 = -\beta^2 \text{ (say)} \quad \text{——} (3)$$

$$\frac{X''}{X} = -\beta^2$$

$$X'' = -\beta^2 X$$

$$X'' + \beta^2 X = 0$$

$$D^2X + \beta^2 X = 0$$

$(D^2+\beta^2)X=0$

Aux. Eqn

$D^2+\beta^2=0$

$D=\pm i\beta$

$D=0\pm i\beta$

$CF=e^{0x}(c_1\cos\beta x+c_2\sin\beta x)$

$CF=c_1\cos\beta x+c_2\sin\beta x$

$PI=0$

$X=CF+PI$

$$\boxed{X=c_1\cos\beta x+c_2\sin\beta x}$$

From (3)

$$\frac{Y'}{Y}+2=-\beta^2$$

$$\frac{Y'+2Y}{Y}=-\beta^2$$

$$Y'+2Y=-\beta^2 Y$$

$$Y' + 2Y + p^2 Y = 0$$

$$DY + 2Y + p^2 Y = 0$$

$$(D + 2 + p^2) Y = 0$$

Aux. Eqn

$$D + 2 + p^2 = 0$$

$$D = -(p^2 + 2)$$

$$CF = C_3 e^{-(p^2+2)y}$$

$$PI = 0$$

$$Y = CF + PI$$

$$\boxed{Y = C_3 e^{-(p^2+2)y}}$$

Put X and Y in (2)

$$\boxed{u = (C_1 \cos px + C_2 \sin px) C_3 e^{-(p^2+2)y}}$$

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Unit-2 : Applications of Partial Differential Equations and Fourier Transform

DPP-1

Topic : Method of separation of variables(When boundary conditions are not given)

Q .1. *Use the method of separation of variables to solve the equation* $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ *UPTU-2012*

Ans. $u(x,\ y) = c_1 c_2\ e^{k(x+y)}$

Q.2. *Use the method of separation of variables to solve the equation* $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$ *UPTU-2007*

Ans. $u(x,\ y) = c_1\ e^{-p^2 y}\ (c_2\ cos\ px + c_3\ sin\ px)$

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