ENGINEERING MATHEMATICS

UNIT-3 : *Statistical Techniques – I*

Lec-3

Today's Target

- ***Relation between μ_r and μ'_r***
- ***Relation between μ_r and ν_r***
- PYQ
- DPP

Moments

- **Moment about mean (μ_r)**
- **Moment about any arbitrary number (μ'_r)**
- **Moment about origin (ν_r)**

$\mu_0 = 1$

$\mu'_0 = 1$

$\nu_0 = 1$

$\mu_1 = 0$

$\mu_1' = \bar{x} - A$

$\nu_1 = \bar{x}$

$$\text{Variance} = \frac{\sum f_i (x_i - \bar{x})^2}{N} = \mu_2$$

$$\text{Variance} = \mu_2$$

$$\text{Variance} = (S.D)^2 = \sigma^2$$

$$S.D = \sqrt{\mu_2}$$

Relation between μ_r and μ'_r

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$$

(1)

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'$

$\mu_3 = \mu_3' - \mu_2' + \mu_1'$

$\mu_4 = \mu_4' - \mu_3' + \mu_2' - \mu_1'$

μ_1' II $\mu_1'^2$ III $\mu_1'^3$ IV

(3) $\mu_1 = 0$ ✓

$\mu_2 = \mu_2' - \mu_1'^2$ ✓

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$ (3 2)

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

(4 6 3)

Relation between μ_r and ν_r

$$\nu_1 = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_3 = \mu_3 + 3\mu_2\,\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + 4\mu_3\,\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4.$$

(1)

$$\nu_1 = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}$$

$$\nu_3 = \mu_3 + \mu_2 + \bar{x}$$

$$\nu_4 = \mu_4 + \mu_3 + \mu_2 + \bar{x}$$

$\bar{x}$ — II

$\bar{x}^2$ — III

$\bar{x}^3$ — IV

(2)

$$\nu_1 = \bar{x} \quad \checkmark$$

$$\nu_2 = \mu_2 + \bar{x}^2 \quad \checkmark$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

KARL PEARSON'S β AND γ COEFFICIENTS

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

→ β-coefficient

$$\gamma_1 = +\sqrt{\beta_1}$$

$$\gamma_2 = \beta_2 - 3$$

→ γ-coefficient

Q.1. *The first three moments of a distribution, about the value 2 of the variable are 1, 16 and −40.*

(i) Show that the mean is 3, variance is 15 and $\mu_3 = -86$.

(ii) Find the values of the first three moments about the origin.

Given

$A = 2$

$\mu_1' = 1$

$\mu_2' = 16$

$\mu_3' = -40$

(i) We know that

$\mu_1' = \bar{x} - A$

$\bar{x} = \mu_1' + A$

$\bar{x} = 1 + 2$

$\boxed{\bar{x} = 3}$

variance $= \mu_2$

$= \mu_2' - \mu_1'^2$

$= 16 - 1^2$

$\boxed{\text{variance} = 15}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1^2$

$= -40 - 3 \times 16 \times 1 + 2 \times 1^2$

$\boxed{\mu_3 = -86}$

(ii) We know that

$$\nu_1 = \bar{x}$$

$$\boxed{\nu_1 = 3}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_2 = 15 + 3^2$$

$$\boxed{\nu_2 = 24}$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_3 = -86 + 3 \times 15 \times 3 + 3^3$$

$$\nu_3 = -86 + 135 + 27$$

$$\boxed{\nu_3 = 76}$$

Q. 2. For a distribution, the mean is 10, variance is 16, γ_1 is 1, and β_2 is 4. Find the first four moments about the origin.

Given

$\bar{x} = 10$

$\mu_2 = 16$

$\gamma_1 = 1$

$\beta_2 = 4$

We know that

$\gamma_1 = +\sqrt{\beta_1}$

$1 = \sqrt{\beta_1}$

$\boxed{\beta_1 = 1}$

$\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$

$1 = \dfrac{\mu_3^2}{(16)^3}$

$\mu_3^2 = 1 \times (16)^3$

$\mu_3 = \sqrt{16 \times 16 \times 16}$

$\mu_3 = 16 \times 4$

$\boxed{\mu_3 = 64}$

Also

$\beta_2 = \dfrac{\mu_4}{\mu_2^2}$

$4 = \dfrac{\mu_4}{16^2}$

$\mu_4 = 256 \times 4$

$\boxed{\mu_4 = 1024}$

$\boxed{\mu_2 = 16}$

$\boxed{\mu_1 = 0}$

Q.3. *The first four moments of a distribution about the value '4' of the variable are* $-1.5, 17, -30$ *and* 108. *Find the moments about mean, about origin,* β_1 *and* β_2. *Also, find the moments about the point* $x = 2$. **(UPTU-2017)**

Given

$A = 4$

$\mu_1' = -1.5$

$\mu_2' = 17$

$\mu_3' = -30$

$\mu_4' = 108$

Moments about Mean

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2 = 17 - (-1.5)^2$

$\mu_2 = 17 - 2.25$

$\mu_2 = 14.75$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1^3$

$\mu_3 = -30 - 3 \times 17 \times (-1.5) + 2(-1.5)^3$

$\mu_3 = 39.75$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 108 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4$$

$$\boxed{\mu_4 = 142.3125}$$

Moments about origin

$$\nu_1 = \bar{x}$$

$$\nu_1 = \mu_1' + A$$

$$\nu_1 = -1.5 + 4$$

$$\boxed{\nu_1 = 2.5}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_2 = 14.75 + (2.5)^2$$

$$\boxed{\nu_2 = 21}$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_3 = 39.75 + 3 \times 14.75 \times 2.5 + (2.5)^3$$

$$\nu_3 = 166$$

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

$$\nu_4 = 142.3125 + 4 \times 39.75 \times 2.5 + 6 \times 14.75 \times (2.5)^2 + (2.5)^4$$

$$\nu_4 = 1132$$

Karl pearson's coefficient

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(39.75)^2}{(14.75)^3} = 0.4923775$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\beta_2 = \frac{142.3125}{(14.75)^2}$$

$$\beta_2 = 0.65412$$

Moment About A = 2

$A = 2$

$\mu_1' = \bar{x} - A$

$\mu_1' = 2.5 - 2$

$\boxed{\mu_1' = 0.5}$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2' = \mu_2 + \mu_1'^2$

$\mu_2' = 14.75 + (0.5)^2$

$\boxed{\mu_2' = 15}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3' = \mu_3 + 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3 = 39.75 + 3 \times 15 \times 0.5 + 2 \times (0.5)^3$

$\boxed{\mu_3 = 62}$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4' = \mu_4 + 4\mu_3'\mu_1' - 6\mu_2'\mu_1'^2 + 3\mu_1'^4$$

$$\mu_4' =$$

$$\boxed{\mu_4' = 244}$$

Q.4. *The first four mements of a distribution about the value* **0** *are* $-0.20, 1.76, -2.36$ *and* 10.88. *Find the moments about the mean.* **(UPTU-2009)**

Given

$A = 0$

$\mu_1' = -0.20$

$\mu_2' = 1.76$

$\mu_3' = -2.36$

$\mu_4' = 10.88$

DPP- 3

Topic : *Relation between μ_r and μ'_r*
Relation between μ_r and ν_r

Q.1. $The\ first\ four\ moments\ of\ a\ distribution\ about\ x = 2\ are\ 1, 2.5, 5.5$ $and\ 16.\ Calcutate\ the\ first\ four\ moments\ about\ the\ mean\ and\ about\ origin.$

Q.2 $In\ a\ certain\ distribution, the\ first\ four\ mements\ about\ the\ point$ $x = 4\ are\ -1.5, 17, -30\ and\ 308.\ Find\ the\ moments\ about\ mean\ and\ about\ origin.$ $Also, calculate\ \beta_1 and\ \beta_2.$ (UPTU-2014)

Q.3. *The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate mean, variance, μ_3 and μ_4.*

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