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DPP - 4 Maths - IV Unit - 2

Topic - Solution of one Dimensional Wave Equation

Q-1 : Find the deflection of the vibrating string which is fixed at the ends $x=0$ and $x=2$ and the motion is started by displacing the string into the form $\sin^3\left(\frac{\pi x}{2}\right)$ and releasing it with zero initial velocity at $t=0$.

Sol : The equation of string is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

Length of string $(l) = 2$

Boundry conditions, At $x=0$, $y=0$ BC-1
At $x=2$, $y=0$ BC-2

Initial conditions, At $t=0$, $\frac{\delta y}{\delta t}=0$ IC-1

At $t=0$, $y = \sin^3\frac{\pi x}{2}$ IC-2

Most Suitable solution

$$y = (C_1 \cos px + C_2 \sin px)(C_3 \cos cpt + C_4 \sin cpt)$$

Apply Boundary Condition-1,

$$0 = C_1 (C_3 \cos cpt + C_4 \sin cpt)$$

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$C_1 = 0$

Updated solution :-

$$y = C_2 \sin px \, (C_3 \cos cpt + C_4 \sin cpt)$$

Apply BC-2

$$0 = C_2 \sin 2p \, (C_3 \cos cpt + C_4 \sin cpt)$$

$$\sin 2p = 0$$
$$2p = n\pi$$
$$p = \frac{n\pi}{2}$$

Updated solution

$$y = C_2 \sin \frac{n\pi x}{2} \left(C_3 \cos \frac{n\pi ct}{2} + C_4 \sin \frac{\pi ct}{2} \right)$$

$$\frac{\delta y}{\delta t} = C_2 \sin \frac{n\pi x}{2} \left(-C_3 \times \frac{n\pi c}{2} \sin \frac{n\pi ct}{2} + C_4 \times \frac{n\pi c}{2} \cos \frac{n\pi ct}{2} \right)$$

Apply IC-1

$$0 = C_2 \sin \frac{n\pi x}{2} \times C_4 \times \frac{n\pi c}{2}$$

$$C_4 = 0$$

Updated solution $\Rightarrow C_2 \sin \frac{n\pi x}{2} \times C_3 \cos \frac{n\pi ct}{2}$

$$y = C_2 C_3 \sin \frac{n\pi x}{2} \cos \frac{n\pi ct}{2}$$

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Most general solution :-

$$y(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2} \cos\frac{n\pi c t}{2}$$

Apply IC-2

$$\sin^3\left(\frac{\pi x}{2}\right) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$$

$$\frac{3}{4}\sin\frac{\pi x}{2} - \frac{1}{4}\sin\frac{3\pi x}{2} = b_1 \sin\frac{\pi x}{2} + b_2 \sin\frac{2\pi x}{2} + b_3 \sin\frac{3\pi x}{2} + \ldots.$$

On comparison -

$$b_1 = \frac{3}{4}, \quad b_3 = -\frac{1}{4}$$

$$\boxed{y(x,t) = \frac{3}{4}\sin\frac{\pi x}{2}\cos\frac{\pi c t}{2} - \frac{1}{4}\sin\frac{3\pi x}{2}\cos\frac{3\pi c t}{2}}$$

Ans

Q-2 :- A string is stretched and fastened to two points distance l apart. Find the displacement $y(x,t)$ at any point at a distance x from one end at time t given that: $\sin y(x,0) = A\sin\left(\frac{2\pi x}{l}\right)$

Sol :- The eq of string is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

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Length of string = l

Boundry conditions ⇒ At $x=0, y=0$ BC-1
At $x=l, y=0$ BC-2

Initial conditions ⇒ At $t=0, \frac{\delta y}{\delta t}=0$ IC-1

At $t=0, y=A\sin\left(\frac{2\pi x}{l}\right)$ IC-2

Most suitable condition solution

$$y=(C_1\cos px+C_2\sin px)(C_3\cos cpt+C_4\sin cpt)$$

Apply BC-1

$$0=C_1(C_3\cos cpt+C_4\sin cpt)$$
$$C_1=0$$

Updated solution ⇒ $y=C_2\sin px(C_3\cos cpt+C_4\sin cpt)$

Apply BC-2

$$0=C_2\sin pl(C_3\cos cpt+C_4\sin cpt)$$
$$\sin pl=0$$
$$pl=n\pi$$
$$p=\frac{n\pi}{l}$$

Updated solution ⇒ $y=C_2\sin\frac{n\pi x}{l}\left(C_3\cos\frac{n\pi ct}{l}+C_4\sin\frac{n\pi ct}{l}\right)$

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$$\frac{\delta y}{\delta t} = C_2 \sin\frac{n\pi x}{l}\left(-C_3 \times \frac{n\pi c}{l} \sin\frac{n\pi c t}{l} + C_4 \times \frac{n\pi c}{l} \cos\frac{n\pi c t}{l}\right)$$

Apply IC-1

$$0 = C_2 \sin\frac{n\pi x}{l} \times C_4 \times \frac{n\pi c}{l}$$

$$C_4 = 0$$

Updated solution

$$y = C_2 \sin\frac{n\pi x}{l} \times C_3 \cos\frac{n\pi c t}{l}$$

$$y = C_2 C_3 \sin\frac{n\pi x}{l} \cos\frac{n\pi c t}{l}$$

Most general solution

$$y(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \cos\frac{n\pi c t}{l}$$

Apply IC-2

$$A \sin\left(\frac{2\pi x}{l}\right) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$A \sin\left(\frac{2\pi x}{l}\right) = b_1 \sin\frac{\pi x}{l} + b_2 \sin\frac{2\pi x}{l} + \cdots$$

On comparison,

$$b_2 = A$$

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$$y(x,t) = b_1 \sin\frac{\pi x}{l}\cos\frac{\pi c t}{l} + b_2 \sin\frac{2\pi x}{l}\cos\frac{2\pi c t}{l}$$

$$\boxed{y(x,t) = A\sin\left(\frac{2\pi x}{l}\right)\cos\left(\frac{2\pi c t}{l}\right)}$$

Ans

Q-3 : A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0 \sin^3\frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x,t)$.

The equation of string is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

Length of string $(l) = l$

Boundry conditions, At $x=0$, $y=0$ BC-1

At $x=l$, $y=0$ BC-2

Initial conditions, At $t=0$, $\frac{\delta y}{\delta t} = 0$ IC-1

At $t=0$, $y = y_0 \sin^3\frac{\pi x}{l}$ IC-2

Most suitable solution,

$$y = (C_1 \cos px + C_2 \sin px)(C_3 \cos cpt + C_4 \sin cpt)$$

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Apply BC-1,

$$0 = C_1 (C_3 \cos cpt + C_4 \sin cpt)$$
$$C_1 = 0$$

Updated solution:-

$$y = C_2 \sin px \ (C_3 \cos cpt + C_4 \sin cpt)$$

Apply BC-2,

$$0 = C_2 \sin p \ (C_3 \cos cpt + C_4 \sin cpt)$$

$$\sin lp = 0$$
$$lp = n\pi$$
$$p = \frac{n\pi}{l}$$

Updated solution,

$$y = C_2 \sin \frac{n\pi x}{l} \left(C_3 \cos c\frac{n\pi}{l} t + C_4 \sin c\frac{n\pi}{l} t\right)$$

$$\frac{\delta y}{\delta t} = C_2 \sin \frac{n\pi x}{l} \left(-C_3 \times \frac{n\pi c}{l} \sin \frac{cn\pi t}{l} + C_4 \times \frac{n\pi c}{l} \cos \frac{n\pi ct}{l}\right)$$

Apply IC-1

$$0 = C_2 \frac{n\pi x}{l} \times C_4 \times \frac{n\pi c}{l}$$

$$C_4 = 0$$

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Updated solution $\Rightarrow y = C_2 \sin \frac{n\pi x}{l} \times C_3 \cos \frac{n\pi ct}{l}$

$$y = C_2 C_3 \sin \frac{n\pi x}{l} \cos \frac{n\pi ct}{l}$$

Most general solution -

$$y(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \cos \frac{n\pi ct}{l}$$

Apply IC - 2

$$y_0 \sin^3 \frac{\pi x}{l} = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$y_0 \left(\frac{3}{4} \sin \frac{\pi x}{l} - \frac{1}{4} \sin \frac{3\pi x}{l} \right) = b_1 \sin \frac{\pi x}{l} + b_2 \sin \frac{2\pi x}{l} + b_3 \sin \frac{3\pi x}{l}$$

On Comparison,

$$b_1 = \frac{3y_0}{4}, \quad b_2 = 0, \quad b_3 = -\frac{y_0}{4}$$

$$\boxed{y(x,t) = \frac{3y_0}{4} \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l} - \frac{y_0}{4} \sin \frac{3\pi x}{l} \cos \frac{3\pi ct}{l}}$$

Ans

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