DPP-1 Maths-IV Unit-2

Topic - Method of seperation of variables

Q-1: Use the method of separation of variables to solve the equation $\frac{\delta u}{\delta x} = \frac{\delta u}{\delta y}$.

Sol- $\frac{\delta u}{\delta x} = \frac{\delta u}{\delta y}$ -(1)

Let $u = X(x)Y(y)$

$u = XY$ -(2)

$\frac{\delta u}{\delta x} = Y\frac{dX}{dx} = YX'$

$\frac{\delta u}{\delta y} = X\frac{dY}{dy} = XY'$

Put these values in equation (1)

$YX' = XY'$

$\frac{X'}{X} = \frac{Y'}{Y} = K$ (say) -(3)

$\frac{X'}{X} = K$

$X' = KX$

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$$X' - KX = 0$$
$$DX - KX = 0$$
$$(D-K)X = 0$$

Auxiliary Equation $\Rightarrow$ $D - K = 0$

$$D = K$$

$$CF = C_1 e^{Kx}$$
$$PI = 0$$

$$X = CF + PI$$

$$\boxed{X = C_1 e^{Kx}}$$

From ③

$$\frac{Y'}{Y} = K$$

$$Y' - YK = 0$$
$$DY - KY = 0$$
$$(D-K)Y = 0$$
$$D = K$$

$$CF = C_2 e^{Ky}$$
$$PI = 0$$

$$\boxed{Y = C_2 e^{Ky}}$$

Put X and Y in eq ②

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$u = C_1 e^{Kx} \cdot C_2 e^{Ky}$

$\boxed{u = C_1 C_2 e^{K(x+y)}}$

Q-2: Use the method of separation of variables to solve the equation $\frac{\delta^2 u}{\delta x^2} - \frac{\delta u}{\delta y} = 0$ — (1)

Let $u = X(x)Y(y)$

$u = XY$ — (2)

$\frac{\delta u}{\delta x} = X'Y$

$\frac{\delta^2 u}{\delta x^2} = X''Y$

$\frac{\delta u}{\delta y} = XY'$

Put these in equation (1)

$X''Y - XY' = 0$

Divide by XY

$\frac{X''}{X} - \frac{Y'}{Y} = 0$

$\frac{X''}{X} = \frac{Y'}{Y} = -p^2$ (say) — (3)

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$$\frac{X''}{X} = -p^2$$

$$X'' = -p^2X$$

$$X'' + p^2X = 0$$

$$D^2X + p^2X = 0$$

$$(D^2 + p^2)X = 0$$

Auxiliary equation $\Rightarrow$ $D^2 + p^2 = 0$

$$D = \pm ip$$

$$C.F. = e^{0x}(C_2 \cos px + C_3 \sin px)$$

$$\boxed{C.F. = C_2 \cos px + C_3 \sin px}$$

$$PI = 0$$

$$X = CF + PI$$

$$\boxed{X = C_2 \cos px + C_3 \sin px}$$

From equation - ③

$$\frac{y'}{y} = -p^2$$

$$y' = -yp^2$$

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$$Y' + Yp^2 = 0$$

$$(D + p^2)Y = 0$$

Auxiliary equation $\Rightarrow$ $D + p^2 = 0$

$$D = -p^2$$

$$\boxed{C.F. = C_1 e^{-p^2 y}}$$

$$PI = 0$$

$$Y = CF + PI$$

$$\boxed{Y = C_1 e^{-p^2 y}}$$

Put values of X and Y in equation (2).

$$u = XY$$

$$\boxed{u = C_1 e^{-p^2 y}(C_2 \cos px + C_3 \sin px)}$$

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