DPP- 1

Topic : *Overview of Measures of central tendency*

Q.1. Find the mean for the following individual series

x	3	6	8	10	18

Q. 2. *Find the mean of the following frequency distribution*

x	0	1	2	3	4	5	6	7
f	1	8	28	56	70	56	28	8

Q.3. *Find the mean of the following frequency distribution*

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	10	40	20	25

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DPP-1 Maths - IV Unit - 3

Topic - Overview of Measures of Central tendency

Q-1: Find the mean for the following individual series.

x	3	6	8	10	18

Sol - $x_1 = 3$
$x_2 = 6$
$x_3 = 8$
$x_4 = 10$
$x_5 = 18$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{3+6+8+10+18}{5}$$

$$\bar{x} = \frac{45}{5}$$

$$\boxed{\bar{x} = 9}$$

Q-2: Find the mean of the following frequency distribution.

x	0	1	2	3	4	5	6	7
f	1	8	28	56	70	56	28	8

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x	f	$f \cdot x$
0	1	0
1	8	8
2	28	56
3	56	168
4	70	280
5	56	280
6	28	168
7	8	56
	$\Sigma f = 255$	$\Sigma f \cdot x = 1016$

By Direct Method :- $\bar{x} = \frac{\Sigma f x}{\Sigma f}$

$$\bar{x} = \frac{1016}{255}$$

$$\boxed{\bar{x} = 3.98}$$

Ans.

Q-3 :- Find the mean of the following frequency distribution

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	10	40	20	25

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Marks	No. of Students	Mid value (x)	$u = \frac{x-A}{h}$	fu
0-10	5	5	-2	-10
10-20	10	15	-1	-10
20-30	40	25 (A)	0	0
30-40	20	35	1	20
40-50	25	45	2	50

$\Sigma f = 100$ $\Sigma fu = 50$

By step deviation Method -

$$\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}$$

$$\bar{x} = 25 + 10 \times \frac{50}{100}$$

$$\boxed{\bar{x} = 30}$$

Ans

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DPP- 2

Topic : *Moments*

Q.1. $Calculate\ \mu_1, \mu_2, \mu_3, \mu_4, for\ the\ series : 4, 7, 10, 13, 16, 19, 22$

Q.2 *Calculate the first four moment about* $x = 15$ *and hence find the moments about the mean of the following distribution :*

x	10	11	12	13	14	15	16	17	18	19	20	21
f	9	36	75	105	116	107	88	66	45	30	18	5

Q.3 Find the first four moments about origin for the following frequency distribution:

$Marks$	0-10	10-20	20-30	30-40	40-50
$No.of\ Students$	5	10	40	20	25

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DPP-2 Maths- IV Unit-3

Topic- moments

Q-1- Calculate $\mu_1, \mu_2, \mu_3, \mu_4$ for the series: 4, 7, 10, 13, 16, 19, 22.

Sol:- $\bar{x} = \frac{\Sigma x}{n}$

$\bar{x} = \frac{91}{7}$

$\bar{x} = 13$

S.No	x	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$
1	4	-9	81	-729	6561
2	7	-6	36	-216	1296
3	10	-3	9	-27	81
4	13	0	0	0	0
5	16	3	9	27	81
6	19	6	36	216	1296
7	22	9	81	729	6561
	$\Sigma x = 91$	$\Sigma(x-\bar{x}) = 0$	$\Sigma(x-\bar{x})^2 = 252$	$\Sigma(x-\bar{x})^3 = 0$	$\Sigma(x-\bar{x})^4 = 15876$

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Moments ⇒ $\boxed{\mu_1 = 0}$

$$\mu_2 = \frac{\Sigma (x-\bar{x})^2}{n}$$

$$\mu_2 = \frac{252}{7}$$

$$\boxed{\mu_2 = 31.5}$$

$$\mu_3 = \frac{\Sigma (x-\bar{x})^3}{n}$$

$$\boxed{\mu_3 = 0}$$

$$\mu_4 = \frac{\Sigma (x-\bar{x})^4}{n}$$

$$= \frac{15876}{7}$$

$$\boxed{\mu_4 = 2268}$$

Q-2: Calculate the first four moment about $x = 15$ and hence find moments about the following distribution

x	10	11	12	13	14	15	16	17	18	19	20	21
f	9	36	75	105	116	107	88	66	45	30	18	5

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x	f	fx	$(x-A)$	$f(x-A)$	$f(x-A)^2$	$f(x-A)^3$	$f(x-A)^4$
10	9	90	-5	-45	225	-1125	5625
11	36	396	-4	-144	576	-2304	9216
12	75	900	-3	-225	675	-2025	6075
13	105	1365	-2	-210	420	-840	1680
14	116	1624	-1	-116	116	-116	116
15	107	1605	0	0	0	0	0
16	88	1408	1	88	88	88	88
17	66	1122	2	132	264	528	1056
18	45	810	3	135	405	1215	3645
19	30	570	4	120	480	1920	7680
20	18	360	5	90	450	2250	11250
21	5	105	6	30	180	1080	6480
	$\Sigma f =$ 700	$\Sigma fx =$ 10,355		$\Sigma f(x-A) = -145$	$\Sigma f(x-A)^2 = 3879$	$\Sigma f(x-A)^3 = 671$	$\Sigma f(x-A)^4 = 52911$

$$\mu_1' = \frac{\Sigma f(x-A)}{\Sigma f} = \frac{-145}{700} = -0.207$$

$$\mu_2' = \frac{\Sigma f(x-A)^2}{\Sigma f} = \frac{3879}{700} = 5.541$$

$$\mu_3' = \frac{\Sigma f(x-A)^3}{\Sigma f} = \frac{671}{700} = 0.958$$

$$\mu_4' = \frac{\Sigma f(x-A)^4}{\Sigma f} = \frac{52911}{700} = 75.587$$

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Moment about mean -

$\boxed{\mu_1 = 0}$

$\mu_2 = \mu_2' - \mu_1'^2$

$= 5.541 - (-0.207)^2$

$= 5.541 - 0.042$

$\boxed{\mu_2 = 5.499}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= 0.958 - 3 \times 5.541 \times (-0.207) + 2 \times (-0.207)^3$

$= 0.958 + 3.44 - 0.0177$

$\boxed{\mu_3 = 4.3023}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 75.587 - 4 \times 0.958 \times (-0.207) + 6 \times 5.541 \times (-0.207)^2 - 3 \times (-0.207)^4$

$\boxed{\mu_4 = 77.76}$

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Q-3- Find the first four moments about origin for the following data.

Marks	0-10	10-20	20-30	30-40	40-50
No. of student	5	10	40	20	25

Marks	f	x	fx	$(x-\bar{x})$	$f(x-\bar{x})$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
0-10	5	5	25	-25	-125	3125	-78125	1953125
10-20	10	15	150	-15	-150	2250	-33750	506250
20-30	40	25	1000	-5	-200	1000	-5000	25000
30-40	20	35	700	5	100	500	2500	12500
40-50	25	45	1125	15	375	5625	84375	1265625
	$\Sigma f = 100$		$\Sigma fx = 3000$		$\Sigma f(x-\bar{x}) = 0$	$\Sigma f(x-\bar{x})^2 = 12,500$	$\Sigma f(x-\bar{x})^3 = -30,000$	$\Sigma f(x-\bar{x})^4 = 3762500$

$$\text{Mean } (\bar{x}) = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{3000}{100}$$

$$\boxed{\bar{x} = 30}$$

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Moment about mean:-

$\mu_1 = 0$

$\mu_2 = \frac{\Sigma f (x-\bar{x})^2}{N}$

$= \frac{12500}{100}$

$\mu_2 = 125$

$\mu_3 = \frac{\Sigma f (x-\bar{x})^3}{N}$

$= -\frac{30,000}{100}$

$\mu_3 = -300$

$\mu_4 = \frac{\Sigma f (x-\bar{x})^4}{N}$

$= \frac{3762500}{100}$

$\mu_4 = 37625$

Moment about origin:-

$\nu_1 = \bar{x} = 30$

$\nu_2 = \mu_2 + \bar{x}^2$

$= 125 + 30 \times 30$

$\nu_2 = 1025$

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$$v_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$v_3 = -300 + 3 \times 125 \times 30 + 30 \times 30 \times 30$$

$$v_3 = -300 + 11250 + 27000$$

$$\boxed{v_3 = 37950}$$

$$v_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

$$v_4 = 37625 + 4 \times (-300) \times 30 + 6 \times 125 \times 30 \times 30 + 810000$$

$$v_4 = 37625 - 36000 + 675000 + 810000$$

$$\boxed{v_4 = 1486625}$$

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DPP- 3

Topic : *Relation between μ_r and μ'_r*
Relation between μ_r and ν_r

Q.1. $The\ first\ four\ moments\ of\ a\ distribution\ about\ x = 2\ are\ 1, 2.5, 5.5$ $and\ 16.\ Calcutate\ the\ first\ four\ moments\ about\ the\ mean\ and\ about\ origin.$

Q.2 $In\ a\ certain\ distribution, the\ first\ four\ mements\ about\ the\ point$ $x = 4\ are\ -1.5, 17, -30\ and\ 308.\ Find\ the\ moments\ about\ mean\ and\ about\ origin.$ $Also, calculate\ \beta_1 and\ \beta_2.$ (UPTU-2014)

Q.3. *The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate mean, variance, μ_3 and μ_4.*

Gateway Classes : 7455 961284

DPP-3 Maths-IV Unit-3

Topic - Relation between μ_r and μ_r'

Q-1: The first four moments of a distribution about $x=2$ are 1, 2·5, 5·5 and 16. Calculate first four moments about the mean and about origin.

Sol: $A = 2$

$\mu_1' = 1$

$\mu_2' = 2.5$

$\mu_3' = 5.5$

$\mu_4' = 16$

Moments about mean $\boxed{\mu_1 = 0}$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2 = 2.5 - 1$

$\boxed{\mu_2 = 1.5}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3 = 5.5 - 3 \times 2.5 \times 1 + 2$

$\boxed{\mu_3 = 0}$

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$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 16 - 4 \times 5.5 \times 1 + 6 \times 2.5 \times 1 - 3$$

$$\mu_4 = 16 - 22 + 15 - 3$$

$$\boxed{\mu_4 = 6}$$

Moments about origin:-

$$\nu_1 = \mu_1' + A$$

$$\nu_1 = 1 + 2$$

$$\boxed{\nu_1 = 3} = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_2 = 1.5 + 9$$

$$\boxed{\nu_2 = 10.5}$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_3 = 0 + 3 \times 1.5 \times 3 + 27$$

$$\boxed{\nu_3 = 40.5}$$

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$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

$$\nu_4 = 6 + 4 \times 0 \times 3 + 6 \times 1.5 \times 9 + 81$$

$$\nu_4 = 6 + 81 + 81$$

$$\boxed{\nu_4 = 168}$$

Q-2: In a certain distribution, the first four moments about the point $x = 4$ are $-1.5, 17, -30$ and 308. Find the moments about mean and about origin also calculate β_1 and β_2.

Sol: $A = 4$

$$\mu_1' = -1.5 \quad \mu_2' = 17 \quad \mu_3' = -30 \quad \mu_4' = 308$$

Moments about mean $\Rightarrow$ $\boxed{\mu_1 = 0}$

$$\mu_2 = \mu_2' - \mu_1^2$$

$$\mu_2 = 17 - (-1.5)^2$$

$$\mu_2 = 17 - 2.25$$

$$\boxed{\mu_2 = 14.75}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_3 = -30 - 3 \times 17 \times -1.5 + 2 \times (-3.375)$$

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$\mu_3 = -30 + 76.5 - 6.75$

$\boxed{\mu_3 = 39.75}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\mu_4 = 308 - 4 \times (-30) \times (-1.5) + 6 \times 17 \times (-1.5)^2 - 3 \times (-1.5)^4$

$\mu_4 = 308 - 180 + 229.5 - 15.18$

$\boxed{\mu_4 = 342.32}$

Moment about origin

$\nu_1 = \mu_1' + A$

$\nu_1 = -1.5 + 4$

$\boxed{\nu_1 = 2.5 = \bar{x}}$

$\nu_2 = \mu_2 + \bar{x}^2$

$\nu_2 = 14.75 + (2.5)^2$

$\boxed{\nu_2 = 21}$

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$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_3 = 39.75 + 3 \times 14.75 \times 2.5 + (2.5)^3$$

$$\boxed{\nu_3 = 165.995}$$

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

$$\nu_4 = 342.32 + 397.5 + 553.125 + 39.062$$

$$\boxed{\nu_4 = 1332}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(39.75)^2}{(14.75)^3} = \frac{1580.062}{3209.04}$$

$$\boxed{\beta_1 = 0.4923}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{342.32}{217.56} = 1.57$$

Q- The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate mean, variance, μ_3 and μ_4.

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Sol:- $A = 5$

$\mu_1' = 2, \quad \mu_2' = 20, \quad \mu_3' = 40, \quad \mu_4' = 50$

$\bar{x} = \mu_1' + A$

$\bar{x} = 2 + 5$

$\boxed{\bar{x} = 7}$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2 = 20 - 4$

$\boxed{\mu_2 = 16 = \text{Variance}}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3 = 40 - 3 \times 20 \times 2 + 2 \times 8$

$\mu_3 = 40 - 120 + 16$

$\boxed{\mu_3 = -64}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\mu_4 = 50 - 4 \times 40 \times 2 + 6 \times 20 \times 4 - 3 \times 16$

$\boxed{\mu_4 = 162}$

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DPP- 4

Topic : *Skewness*

Q.1 (i) Define skewness of a distribution. (UPTU-2015)

(ii) Define the coefficients of skewness. (MTU 2012, 2013)

Q.2. Karl Pearson's coefficient of skewness of a distribution is 0.32, its standard deviation is 6.5 and mean is 29.6. Find the mode of the distribution.

Q.3. The first-three central moments of a distribution are 0, 2.5, 0.7. Find the value of the moment coefficient of skewness. (UPTU-2015)

Q.4 In a certain distribution, the first four moments about the point 4 are – 1.5, 17, - 30 and 308. Calculate the moment coefficient of skewness. (UPTU-2015)

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DPP-4 Maths - IV Unit - 3

Topic- Skewness

Q-1 - (i) Define skewness of a distribution.

Ans- Skewness means lack of symmetry in frequency distribution.

a) Mean = Median = Mode — No Skewness

b) Mean > Mode — Positively skewed

c) Mean < Mode — Negatively skewed

ii) Define the coefficients of skewness.

Ans- Relative measures of skewness are called coefficient of skewness.

$$SK_p = \frac{\text{Mean} - \text{Mode}}{SD}$$

$SK_p = 0$ — Symmetrical

$SK_p > 0$ — Positively skewed

$SK_p < 0$ — Negatively skewed

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Q-2- Karl Pearson's coefficient of skewness of a distribution is 0.32, its standard deviation is 6.5 and mean is 29.6. Find the mode of the distribut

Ans-2- Mean $(\bar{x})$ = 29.6

Standard deviation = 6.5

S_{KP} = 0.32

$$S_{KP} = \frac{\text{Mean} - \text{Mode}}{SD}$$

$$0.32 = \frac{29.6 - \text{Mode}}{6.5}$$

$$29.6 - \text{Mode} = 6.5 \times 0.32$$

$$29.6 - \text{Mode} = 2.08$$

$$\text{Mode} = 29.6 - 2.08$$

$$\boxed{\text{Mode} = 27.52}$$

Q-3 The first three central moments of a distribution are 0, 2.5, 0.7. Find the value of the moment coefficient of skewness.

$\mu_1 = 0$, $\mu_2 = 2.5$, $\mu_3 = 0.7$

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{0.7}{\sqrt{15.625}} = \frac{0.7}{3.95}$$

$\boxed{S_{K_M} = 0.177}$ positively skewed

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Q-4- In a certain distribution, the first four moments about the point 4 are -1·5, 17, -30 and 308. Calculate the moment coefficient of skewness.

Ans-4 - $A = 4$, $\mu_1' = -1.5$, $\mu_2' = 17$, $\mu_3' = -30$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$\mu_2 = 17 - 2.25$$

$$\boxed{\mu_2 = 14.75}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1^3$$

$$= -30 - 3 \times 17 \times (-1.5) + 2 \times (-3.375)$$

$$= -30 + 76.5 - 6.75$$

$$\boxed{\mu_3 = 39.75}$$

$$SK_M = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{39.75}{\sqrt{3209.04}}$$

$$= \frac{39.75}{56.64}$$

$$\boxed{SK_M = 0.7018}$$ positively skewed

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DPP- 4

Topic : *Problems based on Moments, Skewness and Kurtosis*

Q.1 *The first four moments of a distribution about the value '4' of the variable are -1.5, 17, -30 and 108. State whether the distribution is leptokurtic or platykurtic.* *(U.P.T.U.2014)*

Q.2 *The first four moments of a distribution are 0, 2.5, 0.7 and 18.75. Test the kurtosis of the distribution.* *(M.T.U.2013)*

Q.3 *The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moments about the mean and comment upon the skewness and kurtosis of the distribution.* *(G.B.T.U.2011)*

Q.4 *Find the measure of skewness and kurtosis on the basis of moments for the following distribution :*

x	1	3	5	7	9
f	1	4	6	4	1

Q.5 *Calculate the first four moments about the mean for the following frequency distribution and hence find the coefficient of skewness and kurtosis and comment upon the nature of the distribution.* *(U.P.T.U.2014)*

Class – Interval	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	6	8	17	21	15	11	2

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DPP-5 Maths-IV Unit-3

Topic - Kurtosis

Q-1: The first four moments of a distribution about the value '4' of the variable are -1.5, 17, -30 and 108. State whether the distribution is leptokurtic or platykurtic.

Ans- $A = 4,\ \mu_1' = -1.5,\ \mu_2' = 17,\ \mu_3' = -30,\ \mu_4' = 108$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_2 = 17 - (-1.5)^2$$

$$\mu_2 = 14.75$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 108 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4$$

$$\mu_4 = 142.3125$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{142.31}{(14.75)^2} = 0.6541$$

Since $\beta_2 < 3$, distribution is playtykurtic.

Q-2: The first four moments of a distribution are 0, 2.5, 0.7 and 18.75. Test the kurtosis of the distribution.

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$\mu_2 = 2.5, \quad \mu_3 = 0.7, \quad \mu_4 = 18.75$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{18.75}{2.5 \times 2.5}$$

$$\beta_2 = \frac{18.75}{6.25}$$

$$\boxed{\beta_2 = 3}$$

Since $\beta_2 = 3$, distribution is Mesokurtic.

Q-3- The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate the moments about the mean and comment upon the skewness and kurtosis of the distribution.

Ans- $A = 5, \quad \mu_1' = 2, \quad \mu_2' = 20, \quad \mu_3' = 40, \quad \mu_4' = 50$

$$\boxed{\mu_1 = 0}$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_2 = 20 - 4$$

$$\boxed{\mu_2 = 16}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$
$$= 40 - 3 \times 20 \times 2 + 2 \times 8$$

$$\boxed{\mu_3 = -64}$$

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$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 50 - 4 \times 40 \times 2 + 6 \times 20 \times 4 - 3 \times 16$$

$$\boxed{\mu_4 = 162}$$

Coefficient of skewness $= \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-64}{\sqrt{4096}}$

$$= \frac{-64}{64}$$

$$= -1 \quad \text{(Negatively skewed)}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\beta_2 = \frac{162}{256}$$

$$\boxed{\beta_2 = 0.63}$$

Since $\beta_2 < 3$, it is playtykurtic.

Q-4- Find the measures of skewness and kurtosis on the basis of moments for the following distribution.

x	1	3	5	7	9
F	1	4	6	4	1

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x	f	$u = \frac{x-5}{2}$	fu	fu^2	fu^3	fu^4
1	1	-2	-2	4	-8	16
3	4	-1	-4	4	-4	4
A = 5	6	0	0	0	0	0
7	4	1	4	4	4	4
9	1	2	2	4	8	16
	$\Sigma f = 16$		$\Sigma fu = 0$	$\Sigma fu^2 = 16$	$\Sigma fu^3 = 0$	$\Sigma fu^4 = 40$

Moments about 5 -

$$\mu_1' = \frac{\Sigma fu}{N} \times h$$

$$\mu_1' = 0$$

$$\mu_2' = \frac{\Sigma fu^2}{N} \times h^2$$

$$= \frac{16}{5} \times 2 \times 2$$

$$\mu_2' = 6.4 \times 2 = 12.8$$

$$\mu_3' = \frac{\Sigma fu^3}{N} \times h^3$$

$$\boxed{\mu_3' = 0}$$

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$$\mu_4' = \frac{\Sigma f u^4}{N} \times h^4$$

$$\mu_4' = \frac{40}{5} \times 16$$

$$\boxed{\mu_4' = 128}$$

Moments about mean

$$\boxed{\mu_1 = 0}$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\boxed{\mu_2 = 12.8}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\boxed{\mu_3 = 0}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\boxed{\mu_4 = 128}$$

Skewness $(SK_M) = \frac{\mu_3}{\sqrt{\mu_2^3}} = 0$

Kurtosis $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{128}{163.84} = 0.781$

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Q- Calculate the first four moment about the mean for the following frequency distribution and hence find the coefficient of skewness and kurtosis and comment nature of the distribution.

Class - Interval	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	6	8	17	21	15	11	2

CLASS	Frequency	MID Value (x)	$u = \frac{x - 22.5}{5}$	fu	fu^2	fu^3	fu^4
5-10	6	7.5	-3	-18	54	-162	486
10-15	8	12.5	-2	-16	32	-64	128
15-20	17	17.5	-1	-17	17	-17	17
20-25	21	22.5	0	0	0	0	0
25-30	15	27.5	1	15	15	15	15
30-35	11	32.5	2	22	44	88	176
35-40	2	37.5	3	6	18	54	162
	$\Sigma f = 80$			$\Sigma fu = -8$	$\Sigma fu^2 = 180$	$\Sigma fu^3 = -86$	$\Sigma fu^4 = 984$

$$\mu_1' = \frac{\Sigma fu}{N} \times h = \frac{-8}{80} \times 5 = -0.5$$

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$$\mu_2' = \frac{\Sigma f u^2}{N} \times h^2$$

$$= \frac{180}{80} \times 5 \times 5$$

$$\boxed{\mu_2' = 56.25}$$

$$\mu_3' = \frac{\Sigma f u^3}{N} \times h^3$$

$$= \frac{-86}{80} \times 5 \times 5 \times 5$$

$$\boxed{\mu_3' = -134.37}$$

$$\mu_4' = \frac{\Sigma f u^4}{N} \times h^4$$

$$= \frac{984}{80} \times 5 \times 5 \times 5 \times 5$$

$$\mu_4' = 7687.5$$

Moment about mean :-

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 56.25 - 0.25$$

$$\boxed{\mu_2 = 56}$$

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$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= -134.37 - 3 \times 56.25 \times (-0.5) + 2 \times (-0.5)^3$$

$$= -134.37 + 84.375 - 0.25$$

$$= -50.245$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 7687.5 - 4 \times (-134.37)(-0.5) + 6 \times 56.25 \times (-0.5)^2 - 3 \times (-0.5)^4$$

$$\boxed{\mu_4 = 7502.937}$$

Measure of kurtosis $\beta_2 = \frac{\mu_4}{\mu_2^2}$

$$= \frac{7502.937}{56 \times 56}$$

$$= 2.39$$

Hence $\beta_2 < 3$, it is platykurtic.

Coefficient of skewness $\Rightarrow \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-134.37}{419.06}$

$$= -0.3206$$

(Negatively skewed)

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Q.1 *By the method of least squares, find the straight line that best fits the following data:*

x:	1	2	3	4	5
y:	14	27	40	55	68

Q.2 *Using the method of least square, fit a straight line to the following data:*

x	1	2	3	4	5
y	2	4	6	8	10

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DPP-6 Maths - IV Unit - 3

Topic - Curve Fitting

Q- By the method of least squares, find straight line that best fits the following data:-

x:	1	2	3	4	5
y:	14	27	40	55	68

Equation of line

$$y = a + bx \quad - (1)$$

Normal equations

$$\Sigma y = na + b\Sigma x \quad - (2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad - (3)$$

$$n = 5$$

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	55	220	16
5	68	340	25
$\Sigma x = 15$	$\Sigma y = 204$	$\Sigma xy = 748$	$\Sigma x^2 = 55$

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Put Σx, Σy and n in eq (2)

$$204 = 5a + 15b \quad -(4)$$

Now Put Σxy, Σx, and Σx^2 in eq (3)

$$748 = 15a + 55b \quad -(5)$$

Solving eq (4) and (5)

$$612 = 15a + 45b$$
$$748 = 15a + 55b$$
$$- \quad - \quad -$$
$$-136 = -10b$$
$$\boxed{b = 13.6}$$

Put value of b in eq (4)

$$204 = 5a + 15 \times 13.6$$
$$5a = 204 - 204$$
$$\boxed{a = 0}$$

Required eq $\Rightarrow y = a + bx$

$$\boxed{y = 13.6x}$$

Ans

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Q-2:- Using the method of least square, fit a straight line to following data:-

x	1	2	3	4	5
y	2	4	6	8	10

Sol- Equation of line $\Rightarrow$ $y = a + bx$ — (1)

Normal equations $\Rightarrow$

$$\Sigma y = na + b\Sigma x \quad -(2)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad -(3)$$

$n = 5$

x	y	xy	x^2
1	2	2	1
2	4	8	4
3	6	18	9
4	8	32	16
5	10	50	25
$\Sigma x = 15$	$\Sigma y = 30$	$\Sigma xy = 110$	$\Sigma x^2 = 55$

Put Σx, Σy and n in eq (2).

$30 = 5a + 15b \quad -(4)$

Put Σxy, Σx^2 and Σx in eq (3).

$110 = 15a + 55b \quad -(5)$

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Solving eq (4) and (5)

$$90 = 15a + 45b$$

$$110 = 15a + 55b$$

$$-20 = -10b$$

$$\boxed{b = 2}$$

$$30 = 5a + 30$$

$$\boxed{a = 0}$$

Required eq of line $\Rightarrow$ $y = a + bx$

$$\boxed{y = 2x}$$ Ans

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DPP- 7

Topic : :
1. Fitting of Parabola
2. Fitting of Exponential Curve

Q.1 *Using the method of least squares, fit second degree parabola $y = a + bx + cx^2$ for the following data:*

x:	2	3	4	5	6
y:	14	17	20	24	29

Q.2 *Using the method of least square, fit a parabolic curve of second degree to the following data:*

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

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DPP-7 Maths-IV Unit-3

Topic - Curve Fitting

Q-1: Using method of least square, fit second degree parabola $y = a+bx+cx^2$ for following data.

x:	2	3	4	5	6
y:	14	17	20	24	29

Sol- Equation of parabola

$$y = a+bx+cx^2 \quad \text{—(1)}$$

Normal equations-

$$\Sigma y = na+b\Sigma x+c\Sigma x^2 \quad \text{—(2)}$$

$$\Sigma xy = a\Sigma x+b\Sigma x^2+c\Sigma x^3 \quad \text{—(3)}$$

$$\Sigma x^2y = a\Sigma x^2+b\Sigma x^3+c\Sigma x^4 \quad \text{—(4)}$$

$n = 5$

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x	y	xy	x^2	x^3	x^4	x^2y
2	14	28	4	8	16	56
3	17	51	9	27	81	153
4	20	80	16	64	256	320
5	24	120	25	125	625	600
6	29	174	36	216	1296	1044
$\Sigma x = 20$	$\Sigma y = 104$	$\Sigma xy = 453$	$\Sigma x^2 = 90$	$\Sigma x^3 = 440$	$\Sigma x^4 = 2274$	$\Sigma x^2y = 2173$

Put values in normal equations

$$104 = 5a + 20b + 90c$$

$$453 = 20a + 90b + 440c$$

$$2173 = 90a + 440b + 2274c$$

On solving these equations we get,

$$a = 11, \quad b = 0.842, \quad c = 0.357$$

Hence required equation

$$\boxed{y = 11 + 0.842x + 0.357x^2}$$

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Q-2- Using the method of least square, if a parabolic curve of second degree to the following data-

x	0	1	2	3	4
y	1	1·8	1·3	2·5	6·3

Sol:- Equation of parabola -

$$y = a + bx + cx^2 \quad \text{—(1)}$$

Normal equations :- $\Sigma y = na + b\Sigma x + c\Sigma x^2$ —(2)

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{—(3)}$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{—(4)}$$

x	y	xy	x^2	x^3	x^4	x^2y
0	1	0	0	0	0	0
1	1·8	1·8	1	1	1	1·8
2	1·3	2·6	4	8	16	5·2
3	2·5	7·5	9	27	81	22·5
4	6·3	25·2	16	64	256	100·8
$\Sigma x = 10$	$\Sigma y = 12·9$	$\Sigma xy = 37·1$	$\Sigma x^2 = 30$	$\Sigma x^3 = 100$	$\Sigma x^4 = 354$	$\Sigma x^2y = 130·3$

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Put all values in normal equations -

$$12.9 = 5a + 10b + 30c$$

$$37.1 = 10a + 30b + 100c$$

$$130.3 = 30a + 100b + 354c$$

On solving these equations

$$a = 1.42$$
$$b = -1.07$$
$$c = 0.55$$

Hence required equation

$$y = a + bx + cx^2$$

$$\boxed{y = 1.42 - 1.07x + 0.55x^2}$$

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Q.1 *Given the following experimental values:*

x:	0	1	2	3
y:	2	4	10	15

Q.2 *Using method of least squares, derive the normal equations to fit the curve* $y = ax^2 + bx$ *. Hence fit this curve to the following data:*

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

Q.3 *Fit the curve* $pv^Y = k$ *to the following data:*

$p\ (kg/cm^2)$	0.5	1	1.5	2	2.5	3
$v\ (Liters)$	1620	1000	750	620	520	460

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DPP - 8 Maths - IV Unit - 3

Topic - Curve Fitting

Q-1- Given the following experimental values:

x:	0	1	2	3
y:	2	4	10	15

Find by the method of least squares a parabola of the type $y = a + bx^2$.

Sol- $y = a + bx^2$ — (1)

Normal equations-

$$\Sigma y = na + b\Sigma x^2 \quad — (1)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^4 \quad — (2)$$

x	y	x^2	x^2y	x^4
0	2	0	0	0
1	4	1	4	1
2	10	4	40	16
3	15	9	135	81
$\Sigma x = 6$	$\Sigma y = 31$	$\Sigma x^2 = 14$	$\Sigma x^2y = 179$	$\Sigma x^4 = 98$

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$n = 4$

Put values of Σx^2, Σy, $\Sigma x^2 y$ and Σx^4 in normal eq:-

$$31 = 4a + 14b \quad \text{— (4)}$$

$$179 = 14a + 98b \quad \text{— (5)}$$

On solving eq (4) and eq (5) we get,

$$a = 2.71, \quad b = 1.43$$

Hence the required curve is :-

$$\boxed{y = 2.71 + 1.43x^2}$$

Q-2:- Using method of least squares, derive the normal equations to fit the cure $y = ax^2 + bx$, Hence fit this curve to following data:

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

$$y = ax^2 + bx \quad \text{— (1)}$$

Normal equations:-

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3$$

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x	y	xy	x^2	x^2y	x^3	x^4
1	1	1	1	1	1	1
2	1.2	2.4	4	4.8	8	16
3	1.8	5.4	9	16.2	27	81
4	2.5	10	16	40	64	256
5	3.6	18	25	90	125	625
6	4.7	28.2	36	169.2	216	1296
7	6.6	46.2	49	323.4	343	2401
8	9.1	72.8	64	582.4	512	4096
	$\Sigma y = 30.5$	$\Sigma xy = 184$	$\Sigma x^2 = 204$	$\Sigma x^2y = 1227$	$\Sigma x^3 = 1296$	$\Sigma x^4 = 8772$

Put values in normal equations -

$$184 = 1296a + 204b \quad - (4)$$

$$1227 = 8772a + 1296b \quad - (5)$$

On solving eq (4) and (5)

$$a = 0.1077 \qquad b = 0.2171$$

Required curve is

$$\boxed{y = 0.1077x^2 + 0.2171x}$$

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Q-3 : Fit the curve $pv^r = k$ to the following data

p (kg/cm²)	0.5	1	1.5	2	2.5	3
v (liters)	1620	1000	750	620	520	460

Sol - $pv^r = k$ — (1)

$$p = \frac{k}{v^r}$$

$$p = kv^{-r}$$

Taking log both side

$$\log p = \log k - r \log v$$

$Y \quad A \quad B \quad X$

$$\boxed{Y = A + BX}$$

Normal equations

$$\Sigma Y = nA + B\Sigma X \quad -(2)$$

$$\Sigma XY = A\Sigma X + B\Sigma X^2 \quad -(3)$$

$$n = 6$$

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v	p	$X = \log v$	$Y = \log p$	XY	X^2
1620	0.5	3.209	−0.301	−0.965	0.25
1000	1	3	0	0	9
750	1.5	2.87	0.176	0.505	8.23
620	2	2.79	0.301	0.839	7.78
520	2.5	2.71	0.397	1.075	7.34
460	3	2.66	0.477	1.268	7.07
		$\Sigma X = 17.239$	$\Sigma Y = 1.05$	$\Sigma XY = 2.722$	$\Sigma X^2 = 39.67$

Put values in normal equation -

$1.05 = 6A + 17.23B$ — (4)

$2.722 = 17.23A + 39.67B$ — (5)

On solving eq (4) and (5)

$A = 0.089$ $\quad B = 0.029$

$\log k = 0.089$

$k = \text{antilog}(0.089)$

$k = 1.22$

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$-\gamma = 0.029$

$$\boxed{\gamma = -0.029}$$

Put k and γ in the curve.

$$\boxed{PV^{-0.029} = 1.22}$$

Ans

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Topic : Correlation and Coefficient of Correlation

Q. 1 *Find the correlation coefficient between x and y for the following data:*

x	60	34	40	50	45	41	22	43
y	75	32	34	40	45	33	12	30

Q. 2 *Calculate the coefficient of correlation between the following ages of husband* (x) *and wife* (y) *by taking* 30 *and* 28 *as assumed mean in case of x and y respectively:*

x:	24	27	28	28	29	30	32	33	35	35	40
y:	18	20	22	25	22	28	28	30	27	30	32

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DPP-9 Maths-IV Unit-3

Topic- Correlation

Q-1- Find the correlation coefficient between x and y for the following data

x	60	34	40	50	45	41	22	43
y	75	32	34	40	45	33	12	30

$n = 8$

x	y	x^2	y^2	xy
60	75	3600	5625	4500
34	32	1156	1024	1088
40	34	1600	1156	1360
50	40	2500	1600	2000
45	45	2025	2025	2025
41	33	1681	1089	1353
22	12	484	144	264
43	30	1849	900	1290
$\Sigma X = 335$	$\Sigma Y = 301$	$\Sigma x^2 = 14895$	$\Sigma y^2 = 13563$	$\Sigma XY = 13880$

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

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$$r_{xy} = \frac{8 \times 13880 - 335 \times 301}{\sqrt{8 \times 14895 - 112225}\ \sqrt{8 \times 13563 - 90601}}$$

$$r_{xy} = \frac{10205}{11141.526}$$

$$\boxed{r_{xy} = 0.9159}$$

Q- Calculate the coefficient of correlation between the following ages of husband (x) and wife (y) by taking 30 and 28 as assumed mean in case of x and y respectively.

x: 24 27 28 28 29 30 32 33 35 35 40

y: 18 20 22 25 22 28 28 30 27 30 32

	x	y	$d_x = x-30 = u$	$d_y = y-28 = v$	u^2	v^2	uv
	24	18	-6	-10	36	100	60
	27	20	-3	-8	9	64	24
	28	22	-2	-6	4	36	12
	28	25	-2	-3	4	9	6
	29	22	-1	-6	1	36	6
A	30	28	A 0	0	0	0	0
	32	28	2	0	4	0	0
	33	30	3	2	9	4	6
	35	27	5	-1	25	1	-5
	35	30	5	2	25	4	10
	40	32	10	4	100	16	40
Σ			$\Sigma u = 11$	$\Sigma v = -26$	$\Sigma u^2 = 217$	$\Sigma v^2 = 270$	$\Sigma uv = 159$

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$$r_{xy} = \frac{n\Sigma uv - \Sigma u \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\ \sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

$$r_{xy} = \frac{11 \times 159 - 11 \times (-26)}{\sqrt{11 \times 217 - 121}\ \sqrt{11 \times 270 - 676}}$$

$$r_{xy} = \frac{1749 + 286}{\sqrt{2266}\ \sqrt{2294}}$$

$$r_{xy} = \frac{2035}{47.60 \times 47.8}$$

$$r_{xy} = \frac{2035}{2279.56}$$

$$\boxed{r_{xy} = 0.8927}$$ Ans.

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DPP- 10

Topic : Rank Correlation

Q.1 *Obtain the rank correlation co – efficient for the following data*

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

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DPP-10 Maths - IV Unit - 3

Topic - Rank Correlation

Q-1 : Obtain the rank correlation co-efficients of the following data

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

X	R_1	Y	R_2	$D = R_1 - R_2$	D^2
68	4	62	5	-1	1
64	6	58	7	-1	1
75	2.5	68	3.5	-1	1
50	9	45	10	-1	1
64	6	81	1	5	25
80	1	60	6	-5	25
75	2.5	68	3.5	-1	1
40	10	48	9	1	1
55	8	50	8	0	0
64	6	70	2	4	16

$\Sigma D^2 = 72$

$$r = 1 - \frac{6(\Sigma D^2 + F)}{n(n^2-1)} \quad \text{— (1)}$$

$$F = \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \frac{m_3(m_3^2-1)}{12}$$

$75 \rightarrow 2$, $m_1 = 2$

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$64 \rightarrow 3$, $m_2 = 3$
$68 \rightarrow 2$, $m_3 = 2$

$$F = \frac{2(4-1)}{12} + \frac{3(9-1)}{12} + \frac{2(4-1)}{12}$$

$$F = \frac{6}{12} + \frac{24}{12} + \frac{6}{12}$$

$$F = \frac{1}{2} + \frac{2+1}{2}$$

$$F = 3$$

From (1)

$$r = 1 - \frac{6(72+3)}{10(100-1)}$$

$$r = 1 - \frac{6 \times 75}{10 \times 99}$$

$$r = \frac{6}{11}$$

$$\boxed{r = 0.545}$$

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DPP- 11

Topic : *Regression Lines*

Q. 1 *From the data given, find the equation of lines of regression of x on y and y on x. Also, calculate the correlation co − efficient.*

x:	2	4	6	8	10
y:	5	7	9	8	11

Q. 2 *For two random variables, x and y with the same mean, the two regression equations are $y = ax + b$ and $x = \alpha y + \beta$. Show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$.*

Find also the common mean. (AKTU – 2017)

$\bar{x} = \bar{y} = m$

Q. 3 *The following data regarding the heights (y) and weights (x) of 100 college students are given:* (AKTU – 2018)

$$\sum x = 15000, \quad \sum x^2 = 2272500, \quad \sum y = 6800, \quad \sum y^2 = 463025 \text{ and } \sum xy = 1022250$$

Find the equation of regression line of height on weight.

Q. 4 *The equations of two regression lines, obtained in a correlation analysis of 60 observations are:*

$$5x = 6y + 24 \text{ and } 1000y = 768x - 3608.$$

What is the correlation coefficient? Show that the ratio of coefficient of variablity of x to that of y is $\frac{5}{24}$. What is the ratio of x and y. (AKTU-2019)

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DPP-11 Maths-4 Unit-3

Topic- Regression liner

Q-1 - From the data given. Find the equation of liner of regression of x on y and y on x. Calculate correlation co-efficient

x	2	4	6	8	10
y	5	7	9	8	11

Sol-

x	y	x^2	y^2	xy
2	5	4	25	10
4	7	16	49	28
6	9	36	81	54
8	8	64	64	64
10	11	100	121	110
$\Sigma x = 30$	$\Sigma y = 40$	$\Sigma x^2 = 220$	$\Sigma y^2 = 340$	$\Sigma xy = 266$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{5 \times 266 - 30 \times 40}{5 \times 220 - (30)^2}$$

$$= \frac{1330 - 1200}{1100 - 900}$$

$$b_{yx} = \frac{130}{200} = 0.65$$

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$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{266 \times 5 - 30 \times 40}{5 \times 340 - (40)^2}$$

$$= \frac{130}{100}$$

$$\boxed{b_{xy} = 1.3}$$

$$r = \sqrt{b_{xy} \times b_{yx}}$$

$$r = \sqrt{0.65 \times 1.3}$$

$$\boxed{r = 0.919}$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{30}{5} = 6$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{40}{5} = 8$$

Regression line

y on x $\Rightarrow$ $(y - \bar{y}) = b_{yx}(x - \bar{x})$

$$y - 8 = 0.65(x - 6)$$

$$y - 8 = 0.65x - 3.9$$

$$\boxed{y = 0.65x + 4.1}$$

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x on $y \Rightarrow (x-\bar{x}) = b_{xy}(y-\bar{y})$

$x-6 = 1.3(y-8)$

$x-6 = 1.3y - 10.4$

$\boxed{x = 1.3y - 4.4}$

Q-2- For two random variables, x and y with the same mean, two regression equations are $y = ax+b$ and $x = \alpha y + \beta$. Show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$. Find also common mean.

Sol:- $y = ax+b$ — (1)

$b_{yx} = a$

$x = \alpha y + b$ — (2)

$b_{xy} = \alpha$

Let the common mean be m, then regression liner

y on $x \Rightarrow y-m = a(x-m)$

$\boxed{y = ax + m(1-a)}$ — (3)

x on $y \Rightarrow x-m = \alpha(y-m)$

$\boxed{x = \alpha y + m(1-\alpha)}$ — (4)

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Comparing equation (1) and (3) we get,

$$\boxed{b = m(1-a)}$$

Comparing equation (2) and (4) we get,

$$\boxed{\beta = m(1-\alpha)}$$

$$\therefore \boxed{\frac{b}{\beta} = \frac{1-a}{1-\alpha}}$$

Proved

Since regression lines pass through mean,

$\bar{x} = \alpha\bar{y} + \beta$ and $\bar{y} = a\bar{x} + b$

We assume common mean = m

$m = \alpha m + \beta$ and $m = am + b$

$\therefore \alpha m + \beta = am + b$

$\alpha m - am = b - \beta$

$m(\alpha - a) = b - \beta$

$$\boxed{m = \frac{b-\beta}{\alpha-a}}$$

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Q-3- The following data regarding the heights (y) and weights (x) of 100 college students are given-

$\Sigma x = 15000$, $\Sigma x^2 = 2272500$, $\Sigma y = 6800$, $\Sigma y^2 = 463025$
$\Sigma xy = 1022250$

Find the equation of regression line of height on weight.

Sol- $$\bar{x} = \frac{\Sigma x}{n} = \frac{15000}{100}$$

$$\boxed{\bar{x} = 150}$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{6800}{100}$$

$$\boxed{\bar{y} = 68}$$

Regression coefficient of y on x,

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{(100 \times 1022250) - (15000 \times 6800)}{(100 \times 2272500) - (15000)^2} = 0.1$$

Regression line of height (y) on weight (x) is given by

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 68 = 0.1(x - 150)$$
$$\boxed{y = 0.1x + 53}$$ Ans

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Q- The equations of two regression lines, obtained in a correlation analysis of 60 observations are $5x = 6y + 24$ and $1000y = 768x - 3608$

What is the correlation coefficient? Show that the ratio of coefficient of variability of x to y is 5/24. What is the ratio of variance of x and y?

Sol- Regression line of x on y is

$$5x = 6y + 24$$

$$x = \frac{6}{5}y + \frac{24}{5}$$

$$\boxed{b_{xy} = \frac{6}{5}}$$

Regression line of y on x is:-

$$1000y = 768x - 3608$$

$$y = 0.768x - 3.608$$

$$\boxed{b_{yx} = 0.768}$$

We know, $r = \sqrt{b_{xy} \times b_{yx}}$

$$r = \sqrt{1.2 \times 0.768}$$

$$r = \sqrt{0.921}$$

$$\boxed{r = 0.96}$$

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We know,

$$b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

$$\frac{6}{5} = 0.96\,\frac{\sigma_x}{\sigma_y}$$

$$\frac{6}{4.8} = \frac{\sigma_x}{\sigma_y}$$

$$\boxed{\frac{\sigma_x}{\sigma_y} = 1.25}$$

Since mean satisfies the regression lines,

$$5\bar{x} = 6\bar{y} + 24 \quad \text{—①}$$

$$1000\bar{y} = 768\bar{x} - 3608 \quad \text{—②}$$

By solving eq ① and ② we get,

$$\boxed{\bar{x} = 6} \text{ and } \boxed{\bar{y} = 1}$$

Coefficient of variability of $x = \frac{\sigma_x}{\bar{x}}$

Coefficient of variability of $y = \frac{\sigma_y}{\bar{y}}$

$$\text{Required ratio} = \frac{\sigma_x}{\bar{x}} \times \frac{\bar{y}}{\sigma_y} = \frac{\bar{y}}{\bar{x}} \times \frac{\sigma_x}{\sigma_y} = \frac{1}{6} \times \frac{5}{4} = \frac{5}{24}$$

Proved

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