Engineering Mathematics

Unit: Partial Differential Equations (PDE)

Lec-3

Today's Target

- Formation of PDE [Part-2]
- Univ. Questions
- DDPs



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Formation of PDE

By elimination of arbitrary constants

By elimination of arbitrary functions

Note :

$$\frac{\partial z}{\partial x} = p \qquad \frac{\partial z}{\partial y} = q$$

$$\frac{\partial^2 z}{\partial x^2} = r \qquad \frac{\partial^2 z}{\partial x \partial y} = s \qquad \frac{\partial^2 z}{\partial y^2} = t$$

By elimination of arbitrary function

CASE - 1

When given function is of the form of

$$z = f(x, y)$$

When single function is given

NOTE : Order of PDE = 1

Q.1 **Form the partial differential equation by eliminating the arbitrary function from** $z = f(x^2 - y^2)$ **(Sem. 2015)**

$$z = f(x^2 - y^2) \quad \text{—— (1)}$$

Partially Diff. w.r.t x

$$\frac{\partial z}{\partial x} = f'(x^2 - y^2) \times \frac{\partial}{\partial x}(x^2 - y^2)$$

$$\frac{\partial z}{\partial x} = f'(x^2 - y^2) \times (2x - 0)$$

$$p = 2x f'(x^2 - y^2) \quad \text{—— (2)}$$

Partially Diff. w.r.t y

$$\frac{\partial z}{\partial y} = f'(x^2 - y^2) \times \frac{\partial}{\partial y}(x^2 - y^2)$$

$$\frac{\partial z}{\partial y} = f'(x^2 - y^2) \times (0 - 2y)$$

$$q = -2y f'(x^2 - y^2) \quad \text{— (3)}$$

Divide (2) ÷ (3)

$$\frac{p}{q} = \frac{2x \cancel{f'(x^2 - y^2)}}{-2y \cancel{f'(x^2 - y^2)}}$$

$$\frac{p}{q} = \frac{x}{-y}$$

$$qx = -py$$

$$\boxed{qx + py = 0}$$

CASE - 2

When given function is of the form of

$$z = f_1(x, y) + f_2(x, y)$$

Note: Order of PDE > 1

Q.2 ***Form the partial differential equation by eliminating the arbitrary function from the following : $z = f(x + it) + g(x - it)$.*** ***(Sem. 2011)***

$$z = f(x+it) + g(x-it) \quad \text{——①}$$

Partially diff. w.r.t x

$$\frac{\partial z}{\partial x} = f'(x+it)\times 1 + g'(x-it)\times 1$$

Again partially diff. w.r.t x

$$\frac{\partial^2 z}{\partial x^2} = f''(x+it) + g''(x-it)$$

——②

Partially Diff. w.r.t t

$$\frac{\partial z}{\partial t} = f'(x+it)\times i + g'(x-it)\times(-i)$$

Again partially diff. w.r.t t

$$\frac{\partial^2 z}{\partial t^2} = f''(x+it)\times i^2 + g''(x-it)\times i^2$$

$$\frac{\partial^2 z}{\partial t^2} = i^2\left[f''(x+it) + g''(x-it)\right]$$

$$\frac{\partial^2 z}{\partial t^2} = -\left[f''(x+it) + g''(x-it)\right] \quad \text{——③}$$

Adding equation (2) and (3)

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0$$

order = 2

$\therefore$

Q.3 ***Form the partial differential equation by eliminating the arbitrary function from the following : $Z = x + y + f(xy)$.***

$$z = x + y + f(xy) \quad \text{——①}$$

Partially diff. w.r.t x

$$\frac{\partial z}{\partial x} = 1 + 0 + f'(xy)\,x.y$$

$$p - 1 = f'(xy)\,xy \quad -②$$

Partially diff. w.r.t y

$$\frac{\partial z}{\partial y} = 0 + 1 + f'(xy)\,x\,x$$

$$q - 1 = f'(xy)\,xx \quad -③$$

Divide eqn ② by ③

$$\frac{p-1}{q-1} = \frac{f'(xy)\,xy}{f'(xy)\,x\,x}$$

$$\frac{p-1}{q-1} = \frac{y}{x}$$

$$px - x = qy - y$$

$$\boxed{px - qy = x - y}$$

order = 1

Q.4 Form the partial differential equation by eliminating the arbitrary function from the following : $z = \varphi(x) \cdot \Psi(y)$

$$z = \phi(x) \times \psi(y) \quad -(1)$$

Partially diff. w.r.t x

$$\frac{\partial z}{\partial x} = \phi'(x) \times \psi(y) \quad -(2)$$

Partially diff w.r.t y

$$\frac{\partial z}{\partial y} = \phi(x) \times \psi'(y) \quad -(3)$$

Partially diff. eqn (3) w.r.t x

$$\frac{\partial^2 z}{\partial x \partial y} = \phi'(x) \times \psi'(y) \quad -(4)$$

Multiply eqn (2) and (3)

$$\frac{\partial z}{\partial x} \times \frac{\partial z}{\partial y} = \phi'(x) \times \psi(y) \times \phi(x) \times \psi'(y)$$

$$p \times q = \phi'(x) \times \psi'(y) \times \phi(x) \times \psi(y)$$

$$p \times q = \frac{\partial^2 z}{\partial x \partial y} \times z$$

$$\boxed{p \times q = sz}$$

order = 2

CASE - 3

When given function is of the form of $\varphi(u, v) = 0$

$p = \frac{\partial z}{\partial x}$ $q = \frac{\partial z}{\partial y}$

Where u and v are the function of x, y, z

Steps - (1) Given $\varphi(u, v) = 0$ Where $u = u(x, y, z)$ $v = v(x, y, z)$

(2) Then PDE can be formed by $Pp + Qq = R$ ——— (1)

$$P = \frac{\partial(u, v)}{\partial(y, z)} = \begin{vmatrix} \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \end{vmatrix}$$

$$Q = \frac{\partial(u, v)}{\partial(z, x)} = \begin{vmatrix} \frac{\partial u}{\partial z} & \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial x} \end{vmatrix}$$

$$R = \frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

Put P, Q, R in (1) to get required PDE

Q. 5 Formulate the PDE by eliminating the arbitrary function from $\varphi(x^2+y^2, y^2+z^2)=0$.

$$\phi(x^2+y^2, y^2+z^2)=0$$

Where $u = x^2+y^2$

$$v = y^2+z^2$$

Then required PDE is

$$\boxed{Pp+Qq=R} \quad -(1)$$

$$P=\frac{\partial(u,v)}{\partial(y,z)}=\begin{vmatrix} \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \end{vmatrix}$$

$$P=\begin{vmatrix} 2y & 0 \\ 2y & 2z \end{vmatrix}=4yz$$

$$Q=\frac{\partial(u,v)}{\partial(z,x)}=\begin{vmatrix} \frac{\partial u}{\partial z} & \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial x} \end{vmatrix}$$

$$Q=\begin{vmatrix} 0 & 2x \\ 2z & 0 \end{vmatrix}=-4xz$$

$$R=\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$R=\begin{vmatrix} 2x & 2y \\ 0 & 2y \end{vmatrix}=4xy$$

Put P, Q and R in eqn (1)

$$4yzp-4xzq=4xy$$

$$\boxed{yzp-xzq=xy}$$

order = 1

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Thank You