Engineering Mathematics by Gulshan Sir

Unit-2 : Applications of Partial Differential Equations and Fourier Transform

DPP-1

Topic : Method of separation of variables(When boundary conditions are not given)

Q .1. *Use the method of separation of variables to solve the equation* $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ *UPTU-2012*

Ans. $u(x,\ y) = c_1 c_2\ e^{k(x+y)}$

Q.2. *Use the method of separation of variables to solve the equation* $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$ *UPTU-2007*

Ans. $u(x,\ y) = c_1\ e^{-p^2 y}\ (c_2\ cos\ px + c_3\ sin\ px)$

Gateway Classes

DPP-1 Maths-IV Unit-2

Topic - Method of seperation of variables

Q-1: Use the method of separation of variables to solve the equation $\frac{\delta u}{\delta x} = \frac{\delta u}{\delta y}$.

Sol- $\frac{\delta u}{\delta x} = \frac{\delta u}{\delta y}$ -(1)

Let $u = X(x)Y(y)$

$u = XY$ -(2)

$\frac{\delta u}{\delta x} = Y\frac{dX}{dx} = YX'$

$\frac{\delta u}{\delta y} = X\frac{dY}{dy} = XY'$

Put these values in equation (1)

$YX' = XY'$

$\frac{X'}{X} = \frac{Y'}{Y} = K$ (say) -(3)

$\frac{X'}{X} = K$

$X' = KX$

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$$X' - KX = 0$$
$$DX - KX = 0$$
$$(D-K)X = 0$$

Auxiliary Equation $\Rightarrow$ $D - K = 0$

$$D = K$$

$$CF = C_1 e^{Kx}$$
$$PI = 0$$

$$X = CF + PI$$

$$\boxed{X = C_1 e^{Kx}}$$

From ③

$$\frac{y'}{y} = K$$

$$y' - yK = 0$$
$$Dy - Ky = 0$$
$$(D-K)y = 0$$
$$D = K$$

$$CF = C_2 e^{Ky}$$
$$PI = 0$$

$$\boxed{y = C_2 e^{Ky}}$$

Put X and Y in eq ②

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$u = C_1 e^{Kx} \cdot C_2 e^{Ky}$

$\boxed{u = C_1 C_2 e^{K(x+y)}}$

Q-2: Use the method of separation of variables to solve the equation $\frac{\delta^2 u}{\delta x^2} - \frac{\delta u}{\delta y} = 0$ — (1)

Let $u = X(x)Y(y)$

$u = XY$ — (2)

$\frac{\delta u}{\delta x} = X'Y$

$\frac{\delta^2 u}{\delta x^2} = X''Y$

$\frac{\delta u}{\delta y} = XY'$

Put these in equation (1)

$X''Y - XY' = 0$

Divide by XY

$\frac{X''}{X} - \frac{Y'}{Y} = 0$

$\frac{X''}{X} = \frac{Y'}{Y} = -p^2$ (say) — (3)

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$\frac{X''}{X} = -p^2$

$X'' = -p^2 X$

$X'' + p^2 X = 0$

$D^2 X + p^2 X = 0$

$(D^2 + p^2) X = 0$

Auxiliary equation $\Rightarrow$ $D^2 + p^2 = 0$

$D = \pm ip$

$C.F. = e^{0x}(C_2 \cos px + C_3 \sin px)$

$\boxed{C.F. = C_2 \cos px + C_3 \sin px}$

$PI = 0$

$X = CF + PI$

$\boxed{X = C_2 \cos px + C_3 \sin px}$

From equation - ③

$\frac{y'}{y} = -p^2$

$y' = -yp^2$

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$y' + yp^2 = 0$

$(D + p^2)y = 0$

Auxiliary equation $\Rightarrow$ $D + p^2 = 0$

$D = -p^2$

$$\boxed{C.F. = C_1 e^{-p^2 y}}$$

$PI = 0$

$Y = CF + PI$

$$\boxed{Y = C_1 e^{-p^2 y}}$$

Put values of X and Y in equation (2).

$u = XY$

$$\boxed{u = C_1 e^{-p^2 y}(C_2 \cos px + C_3 \sin px)}$$

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➢ Topic : Method of Separation of Variables (When Boundary conditions are given)

Q .1 *Solve by the method of separation of variables* $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0;\ u(x,\ 0) = 4e^{-x}$ *(AKTU-2017)*

Ans. $u(x,\ y) = 4e^{-x+3y/2}$

Q .2 *Solve by the method of separation of variables* $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u;\ u(0,\ y) = 4e^{-y} - e^{-5y}$ *(AKTU-2022)*

Ans. $u(x,\ y) = 4e^{+x-y} - e^{2x-5y}$

Q .3 *Use the method of separation of variables to solve the equation* $\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$ *given that* $v = 0$ *when* $t \to \infty$ *as well as* $v = 0$ *at* $x = 0$ *and* $x = l$. *(GBTU-2013)*

Ans. $u(x,\ y) = (c_1 \cos px + c_2 \sin px)c_3 e^{-(p^2+2)y}$

Q .4 *Solve the P.D.E. by separation of variables method* $u_{xx} = u_y + 2u,\ u(0,\ y) = 0, \frac{\partial}{\partial x} u(0,\ y) = 1 + e^{-3y}$. *(GBTU-2013)*

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DPP-2 Maths-IV Unit-2

Topic:- Method of separation of Variables

Q-1:- Solve by the method of separation of variables.
$3\frac{\delta u}{\delta x} + 2\frac{\delta u}{\delta y} = 0$; $u(x,0) = 4e^{-x}$.

Sol:- $3\frac{\delta u}{\delta x} + 2\frac{\delta u}{\delta y} = 0$ — (1)

Let $u = X(x)Y(y)$
$u = XY$ — (2)

$\frac{\delta u}{\delta x} = X'Y$

$\frac{\delta u}{\delta y} = XY'$

Put in eq (1)

$3X'Y + 2XY' = 0$

Now divide by XY

$3\frac{X'}{X} + 2\frac{Y'}{Y} = 0$

$3\frac{X'}{X} = -2\frac{Y'}{Y} = K$ (say) — (3)

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$$\frac{3X'}{X} = K$$

$$3X' = XK$$

$$(3D - K)X = 0$$

Auxiliary equation $\Rightarrow$ $3D - K = 0$

$$D = \frac{K}{3}$$

$$\boxed{C.F. = C_1 e^{Kx/3}}$$

$$PI = 0$$

$$X = C.F + PI$$

$$\boxed{X = C_1 e^{Kx/3}}$$

From eq 3.

$$\frac{-2Y'}{Y} = K$$

$$-2Y' = YK$$

$$YK + 2Y' = 0$$

$$(2D + K)Y = 0$$

Auxiliary eq $\Rightarrow$ $2D + K = 0$

$$D = -K/2$$

$$\boxed{C.F. = C_2 e^{-Ky/2}}$$

$$PI = 0$$

$$\boxed{Y = C_2 e^{-Ky/2}}$$

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Put value of X and Y in eq 2:

$u = XY$

$u = C_1 e^{Kx/3} C_2 e^{-Ky/2}$

$$u = C_1 C_2 e^{K\left(\frac{x}{3} - \frac{y}{2}\right)} \quad \text{— (4)}$$

(x, y)

Apply Boundry condition $y = 0$ $\quad u = 4e^{-x}$

$4e^{-x} = C_1 C_2 e^{Kx/3}$

where

$C_1 C_2 = 4$

$-1 = \frac{K}{3}$

$K = -3$

Put $C_1 C_2$ and K in eq — (4)

$u(x, y) = 4e^{-3\left(\frac{x}{3} - \frac{y}{2}\right)}$

$$u(x, y) = 4e^{-x + 3y/2}$$

Ans

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Q-2 : Solve by the method of separation of variables.
$4\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} = 3u$, $u(0,y) = 4e^{-y} - e^{-5y}$

Sol : $4\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} = 3u$ — (1)

Let $u = X(x)Y(y)$

$u = XY$ — (2)

$\frac{\delta u}{\delta x} = X'Y$

$\frac{\delta u}{\delta y} = XY'$

Put value in eq (1)

$4X'Y + XY' = 3XY$

$4\frac{X'}{X} = 3 - \frac{Y'}{Y} = K$ (say)

$4\frac{X'}{X} = K$

$4X' = KX$

$(4D - K)X = 0$

$D = K/4$

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$$\boxed{C.F. = C_1 e^{Kx/4}}$$

$$P.I = 0$$

$$\boxed{X = C_1 e^{Kx/4}}$$

$$3 - \frac{y'}{y} = K$$

$$3y - y' = Ky$$

$$y' + Ky - 3y = 0$$

$$(D + K - 3)y = 0$$

$$D = 3 - K$$

$$\boxed{C.F. = C_2 e^{(3-K)y}}$$

$$P.I = 0$$

$$\boxed{y = C_2 e^{(3-K)y}}$$

From equation (2)

$$u = C_1 C_2 e^{\frac{Kx}{4} + (3-K)y}$$

$$u(x,y) = \sum_{n=1}^{\infty} b_n e^{Kx/4 + (3-K)y} \longrightarrow (3)$$

Apply Boundry conditions: Put $x = 0$

$$4e^{-y} - e^{-5y} = \sum_{n=1}^{\infty} b_n e^{(3-K)y}$$

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$4e^{-y} - e^{-5y} = b_1 e^{(3-K)y} + b_2 e^{(3-K)y} + \ldots\ldots\ldots$

By comparison

$b_1 = 4$ $\quad$ $b_2 = -1$

$-1 = 3-K$ $\quad$ $-5 = 3-K$

$+4 = +K$ $\quad$ $K = 8$

$K = 4$

From ③

$u(x,y) = b_1 e^{\frac{Kx}{4}+(3-K)y} + b_2 e^{\frac{Kx}{4}+(3-K)y}$

$\boxed{u(x,y) = 4e^{x-y} - e^{2x-5y}}$ Ans

Q-3- Use the method of separation of variables to solve the equation $\frac{\delta^2 v}{\delta x^2} = \frac{\delta v}{\delta t}$ given that $v = 0$ when $t \to \infty$ as well as $v = 0$ at $x = 0$ and $x = l$.

Sol- $\frac{\delta^2 v}{\delta x^2} = \frac{\delta v}{\delta t}$ — ①

Let $v = X(x)T(t)$ —

$v = XT$ — ②

$\frac{\delta v}{\delta t} = XT'$, $\frac{\delta v}{\delta x} = X'T$, $\frac{\delta^2 v}{\delta x^2} = X''T$

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Put in eq (1)

$$X''T = XT'$$

Divide by XT

$$\frac{X''}{X} = \frac{T'}{T} = -p^2 \text{ (say)}$$

$$\frac{X''}{X} = -p^2$$

$$X'' = -Xp^2$$

$$X'' + Xp^2 = 0$$

$$(D^2 + p^2)X = 0$$

$$D^2 = -p^2$$
$$D = \pm ip$$

$$C.F. = e^{0x}(C_1 \cos px + C_2 \sin px)$$

$$\boxed{C.F. = C_1 \cos px + C_2 \sin px}$$

$$P.I = 0$$

$$\boxed{X = C_1 \cos px + C_2 \sin px}$$

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$$\frac{T'}{T} = -p^2$$

$$T' = -Tp^2$$

$$T' + Tp^2 = 0$$

$$(D + p^2)T = 0$$

$$D = -p^2$$

$$\boxed{C.F. = C_3 e^{-p^2 t}}$$

$$PI = 0$$

$$\boxed{T = C_3 e^{-p^2 t}}$$

$$v = (C_1 \cos px + C_2 \sin px) C_3 e^{-p^2 t} \quad \text{—③}$$

Apply Boundry conditions,

$v = 0$, $x = 0$

$$0 = C_1 C_3 e^{-p^2 t}$$

$$C_3 \neq 0$$

$$\boxed{C_1 = 0}$$

Now put in eq 3

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Updated solution :-

$$v = C_2 C_3 \sin px \, e^{-p^2 t}$$

Now $v = 0$, $x = l$

$$0 = C_2 C_3 \sin pl \, e^{-p^2 t}$$

$$\sin pl = 0$$

$$pl = n\pi$$

$$\boxed{p = \frac{n\pi}{l}}$$

Updated solution :-

$$v = C_2 C_3 \sin \frac{n\pi}{l} x \, e^{-\frac{n^2\pi^2}{l^2} t}$$

General solution will be :-

$$\boxed{v = \sum_{n=1}^{\infty} b_n e^{-\frac{n^2\pi^2}{l^2} t} \sin \frac{n\pi x}{l}}$$

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Q-4- Solve the P.D.E by separation of variables method,

$$u_{xx} = u_y + 2u \;,\; u(0,y) = 0,\; \frac{\delta}{\delta x} u(0,y) = 1 + e^{-3y}$$

Sol:- Let $u = XY$

$$\frac{\delta u}{\delta y} = XY' \;,\; \frac{\delta u}{\delta x} = X'Y \;,\; \frac{\delta^2 u}{\delta x^2} = X''Y$$

$$u_{xx} = u_y + 2u$$

$$\frac{\delta^2 u}{\delta x^2} = \frac{\delta u}{\delta y} + 2u \qquad \text{— (1)}$$

$$X''Y = XY' + 2XY$$

Divide by XY

$$\frac{X''}{X} = \frac{Y'}{Y} + 2 = k \text{ (say)}$$

$$\frac{X''}{X} = k$$

$$X'' - kX = 0$$
$$(D^2 - k)X = 0$$

Auxiliary Equation $\Rightarrow$ $D^2 - k = 0$

$$D = \pm\sqrt{k}$$

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$$C.F. = C_1 e^{\sqrt{k}x} + C_2 e^{-\sqrt{k}x}$$

$$PI = 0$$

$$\boxed{X = C_1 e^{\sqrt{k}x} + C_2 e^{-\sqrt{k}x}}$$

$$\frac{y'}{y} + 2 = k$$

$$\frac{y'}{y} = k-2$$

$$y' - y(k-2) = 0$$

$$[D-(k-2)]y = 0$$

Auxiliary Equation $\Rightarrow$ $D-(k-2) = 0$

$$D = k-2$$

$$C.F. = C_3 e^{(k-2)y}$$

$$PI = 0$$

$$\boxed{Y = C_3 e^{(k-2)y}}$$

$$u(x,y) = \left(C_1 e^{\sqrt{k}x} + C_2 e^{-\sqrt{k}x}\right) C_3 e^{(k-2)y}$$

Apply Boundry Condition, $x = 0$, $u = 0$.

$$0 = (C_1 + C_2) C_3 e^{(k-2)y}$$

$$C_1 + C_2 = 0$$

$$C_2 = -C_1$$

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Updated solution :

$$u(x,y) = C_1 C_3 \left(e^{\sqrt{k}x} - e^{-\sqrt{k}x}\right) e^{(k-2)y}$$

$$\frac{\delta u}{\delta x} = C_1 C_3 \sqrt{k} \left(e^{\sqrt{k}x} - e^{-\sqrt{k}x}\right) e^{(k-2)y}$$

Put Boundary Condition, $x = 0$

$$\left(\frac{\delta u}{\delta x}\right)_{x=0} = C_1 C_3 \sqrt{k}\,(2)\, e^{(k-2)y}$$

$$1 + e^{-3y} = \Sigma b_n e^{(k-2)y} \qquad \{ b_n = 2 C_1 C_3 \sqrt{k} \}$$

Comparing the coefficients,

i) $b_1 = 1$, $k - 2 = 0$

$k = 2$

$$2 C_1 C_3 \sqrt{k} = 1$$

$$C_1 C_3 = \frac{1}{2\sqrt{2}}$$

ii) $b_3 = -1$, $k - 2 = -3$

$k = -1$

$$2 C_1 C_3 \sqrt{k} = 1$$

$$C_1 C_3 = \frac{1}{2i}$$

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$$u(x,y) = \frac{1}{2\sqrt{2}}\left(e^{\sqrt{2}x} - e^{-\sqrt{2}x}\right) + \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)e^{-3y}$$

$$\boxed{u(x,y) = \frac{1}{\sqrt{2}} \sinh\sqrt{2}\,x + e^{-3y}\sin x}$$

Ans

$\therefore \frac{e^{ax} - e^{-ax}}{2} = \sinh ax$

$\therefore \frac{e^{ix} - e^{-ix}}{2i} = \sin x$

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DPP- 3

Topic : Solution of one Dimensional Wave Equation

Q.1. Derive one dimensional wave equation.

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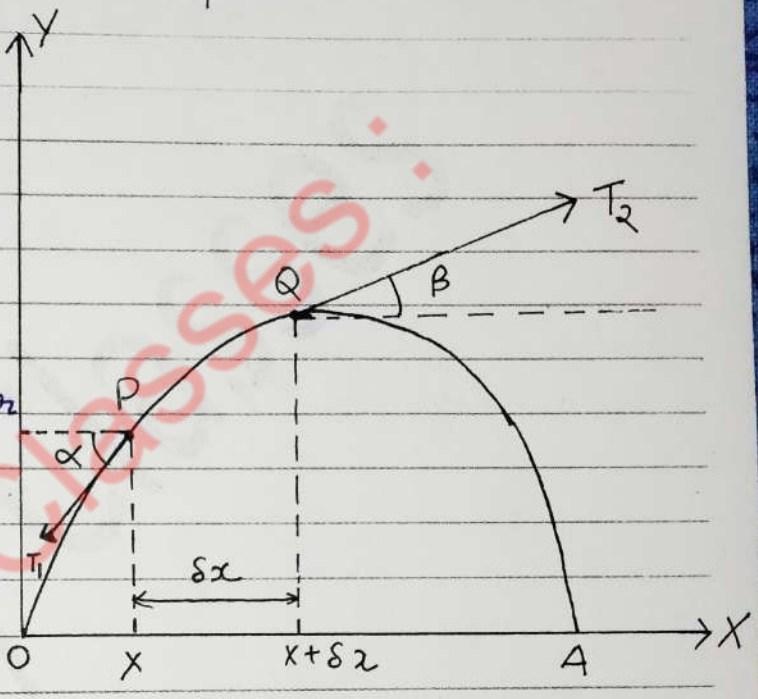
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DPP-3 Maths - IV Unit - 2

Topic - Solution of one Dimensional Wave Equation

Que: Derive one dimensional wave equation.

Sol: Consider a uniform elastic string of length l stretched tightly between two points O and A and displaced slightly from its equilibrium position OA. Taking the O as the origin, OA as the x axis and a perpendicular line through O as the y-axis, we shall find the displacement y as a function of a distance x and the time t.



Let m be the mass per unit length of the string. Consider the motion of an element PQ of length δs. Since the string doesn't offer resistance to bending, the tensions T_1 and T_2 at P and Q respectively are tangential to the curve.

Since, there is no motion in the horizontal direction, we have

$$T_1 \cos\alpha = T_2 \cos\beta = T \text{ (constant)}$$

Mass of element PQ is $m\delta s$. By Newton's second law of motion, the equation of motion in the vertical direction is -

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$$m\delta s \frac{\delta^2 y}{\delta t^2} = T_2 \sin\beta - T_1 \sin\alpha$$

$$\frac{m\delta s}{T} \frac{\delta^2 y}{\delta t^2} = \frac{T_2 \sin\beta}{T_2 \cos\beta} - \frac{T_1 \sin\alpha}{T_1 \cos\alpha} \qquad \text{From eq(1)}$$

$$\frac{\delta^2 y}{\delta t^2} = \frac{T}{m\delta s} (\tan\beta - \tan\alpha)$$

$$\frac{\delta^2 y}{\delta t^2} = \frac{T}{m\delta x} \left[\left(\frac{\delta y}{\delta x}\right)_{x+\delta x} - \left(\frac{\delta y}{\delta x}\right)_x \right]$$

[Since $\delta s = \delta x$ to a first approximation and $\tan\alpha$ and $\tan\beta$ are the slopes of the curve of string at x and $x+\delta x$]

$$\frac{\delta^2 y}{\delta t^2} = \frac{T}{m} \left[\frac{\left(\frac{\delta y}{\delta x}\right)_{x+\delta x} - \left(\frac{\delta y}{\delta x}\right)_x}{\delta x} \right]$$

$$\frac{\delta^2 y}{\delta t^2} = \frac{T}{m} \frac{\delta^2 y}{\delta x^2}, \qquad \text{as } \delta x \to 0$$

$$\boxed{\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}} \qquad \text{where } c^2 = \frac{T}{m}$$

This is the partial differtial equation giving the transverse vibrations of the string. It is also called the one dimmensional wave equation.

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DPP- 4

Topic : Solution of one Dimensional Wave Equation (Part -2)

Q.1. Find the deflection of the vibrating string which is fixed at the ends $x = 0$ and $x = 2$ and the motion is started by displacing the string into the form $sin^3\left(\frac{\pi x}{2}\right)$ and releasing it with zero initial velocity at $t = 0$ *(U.P.T.U. 2014)*

Q.2 A string *is* stretched and fastened to two points distance l apart. Find the displacement $y(x, t)$ at any point at a distance x from one end at time t given that:

$$siny(x, 0) = A\, sin\left(\frac{2\pi x}{l}\right)$$

(M.T.U. 2013)

Q.3 A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0\, sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x, t)$.

(GBTU-2011,2020)

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DPP - 4 Maths - IV Unit - 2

Topic - Solution of one Dimensional Wave Equation

Q-1 : Find the deflection of the vibrating string which is fixed at the ends $x=0$ and $x=2$ and the motion is started by displacing the string into the form $\sin^3\left(\frac{\pi x}{2}\right)$ and releasing it with zero initial velocity at $t=0$.

Sol : The equation of string is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

Length of string $(l) = 2$

Boundry conditions, At $x=0$, $y=0$ BC-1

At $x=2$, $y=0$ BC-2

Initial conditions, At $t=0$, $\frac{\delta y}{\delta t}=0$ IC-1

At $t=0$, $y = \sin^3\frac{\pi x}{2}$ IC-2

Most Suitable solution

$$y = (C_1 \cos px + C_2 \sin px)(C_3 \cos cpt + C_4 \sin cpt)$$

Apply Boundary Condition-1,

$$0 = C_1 (C_3 \cos cpt + C_4 \sin cpt)$$

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$C_1 = 0$

Updated solution :-

$$y = C_2 \sin px \, (C_3 \cos cpt + C_4 \sin cpt)$$

Apply BC-2

$$0 = C_2 \sin 2p \, (C_3 \cos cpt + C_4 \sin cpt)$$

$$\sin 2p = 0$$
$$2p = n\pi$$
$$p = \frac{n\pi}{2}$$

Updated solution

$$y = C_2 \sin \frac{n\pi x}{2} \left(C_3 \cos \frac{n\pi ct}{2} + C_4 \sin \frac{\pi ct}{2}\right)$$

$$\frac{\delta y}{\delta t} = C_2 \sin \frac{n\pi x}{2} \left(-C_3 \times \frac{n\pi c}{2} \sin \frac{n\pi ct}{2} + C_4 \times \frac{n\pi c}{2} \cos \frac{n\pi ct}{2}\right)$$

Apply IC-1

$$0 = C_2 \sin \frac{n\pi x}{2} \times C_4 \times \frac{n\pi c}{2}$$

$$C_4 = 0$$

Updated solution $\Rightarrow C_2 \sin \frac{n\pi x}{2} \times C_3 \cos \frac{n\pi ct}{2}$

$$y = C_2 C_3 \sin \frac{n\pi x}{2} \cos \frac{n\pi ct}{2}$$

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Most general solution :-

$$y(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2} \cos\frac{n\pi c t}{2}$$

Apply IC-2

$$\sin^3\left(\frac{\pi x}{2}\right) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$$

$$\frac{3}{4}\sin\frac{\pi x}{2} - \frac{1}{4}\sin\frac{3\pi x}{2} = b_1 \sin\frac{\pi x}{2} + b_2 \sin\frac{2\pi x}{2} + b_3 \sin\frac{3\pi x}{2} + \ldots.$$

On comparison -

$$b_1 = \frac{3}{4}, \quad b_3 = -\frac{1}{4}$$

$$y(x,t) = \frac{3}{4}\sin\frac{\pi x}{2}\cos\frac{\pi c t}{2} - \frac{1}{4}\sin\frac{3\pi x}{2}\cos\frac{3\pi c t}{2}$$

Ans

Q-2 :- A string is stretched and fastened to two points distance l apart. Find the displacement $y(x,t)$ at any point at a distance x from one end at time t given that: $\text{siny}(x,0) = A\sin\left(\frac{2\pi x}{l}\right)$

Sol :- The eq of string is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

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Length of string = l

Boundry conditions ⇒ At $x=0, y=0$ BC-1
At $x=l, y=0$ BC-2

Initial conditions ⇒ At $t=0, \frac{\delta y}{\delta t}=0$ IC-1

At $t=0, y=A\sin\left(\frac{2\pi x}{l}\right)$ IC-2

Most suitable condition solution

$$y=(C_1\cos px+C_2\sin px)(C_3\cos cpt+C_4\sin cpt)$$

Apply BC-1

$$0=C_1(C_3\cos cpt+C_4\sin cpt)$$
$$C_1=0$$

Updated solution ⇒ $y=C_2\sin px(C_3\cos cpt+C_4\sin cpt)$

Apply BC-2

$$0=C_2\sin pl(C_3\cos cpt+C_4\sin cpt)$$
$$\sin pl=0$$
$$pl=n\pi$$
$$p=\frac{n\pi}{l}$$

Updated solution ⇒ $y=C_2\sin\frac{n\pi x}{l}\left(C_3\cos\frac{n\pi ct}{l}+C_4\sin\frac{n\pi ct}{l}\right)$

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$$\frac{\delta y}{\delta t} = C_2 \sin\frac{n\pi x}{l}\left(-C_3 \times \frac{n\pi c}{l} \sin\frac{n\pi c t}{l} + C_4 \times \frac{n\pi c}{l} \cos\frac{n\pi c t}{l}\right)$$

Apply IC-1

$$0 = C_2 \sin\frac{n\pi x}{l} \times C_4 \times \frac{n\pi c}{l}$$

$$C_4 = 0$$

Updated solution

$$y = C_2 \sin\frac{n\pi x}{l} \times C_3 \cos\frac{n\pi c t}{l}$$

$$y = C_2 C_3 \sin\frac{n\pi x}{l} \cos\frac{n\pi c t}{l}$$

Most general solution

$$y(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \cos\frac{n\pi c t}{l}$$

Apply IC-2

$$A \sin\left(\frac{2\pi x}{l}\right) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$A \sin\left(\frac{2\pi x}{l}\right) = b_1 \sin\frac{\pi x}{l} + b_2 \sin\frac{2\pi x}{l} + \cdots$$

On comparison,

$$b_2 = A$$

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$$y(x,t) = b_1 \sin\frac{\pi x}{l}\cos\frac{\pi c t}{l} + b_2 \sin\frac{2\pi x}{l}\cos\frac{2\pi c t}{l}$$

$$\boxed{y(x,t) = A\sin\left(\frac{2\pi x}{l}\right)\cos\left(\frac{2\pi c t}{l}\right)}$$

Ans

Q-3 : A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0 \sin^3\frac{\pi x}{l}$. If it is released from rest from this position; find the displacement $y(x,t)$.

The equation of string is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

Length of string $(l) = l$

Boundry conditions, At $x=0$, $y=0$ BC-1

At $x=l$, $y=0$ BC-2

Initial conditions, At $t=0$, $\frac{\delta y}{\delta t} = 0$ IC-1

At $t=0$, $y = y_0 \sin^3\frac{\pi x}{l}$ IC-2

Most suitable solution,

$$y = (C_1 \cos px + C_2 \sin px)(C_3 \cos cpt + C_4 \sin cpt)$$

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Apply BC-1,

$$0 = C_1 (C_3 \cos cpt + C_4 \sin cpt)$$
$$C_1 = 0$$

Updated solution :-

$$y = C_2 \sin px \ (C_3 \cos cpt + C_4 \sin cpt)$$

Apply BC-2,

$$0 = C_2 \sin p \ (C_3 \cos cpt + C_4 \sin cpt)$$

$$\sin lp = 0$$
$$lp = n\pi$$
$$p = \frac{n\pi}{l}$$

Updated solution,

$$y = C_2 \sin \frac{n\pi x}{l} \left(C_3 \cos c\frac{n\pi}{l} t + C_4 \sin c\frac{n\pi}{l} t\right)$$

$$\frac{\delta y}{\delta t} = C_2 \sin \frac{n\pi x}{l} \left(-C_3 \times \frac{n\pi c}{l} \sin \frac{cn\pi t}{l} + C_4 \times \frac{n\pi c}{l} \cos \frac{n\pi ct}{l}\right)$$

Apply IC-1

$$0 = C_2 \frac{n\pi x}{l} \times C_4 \times \frac{n\pi c}{l}$$

$$C_4 = 0$$

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Updated solution $\Rightarrow y = C_2 \sin \frac{n\pi x}{l} \times C_3 \cos \frac{n\pi c t}{l}$

$$y = C_2 C_3 \sin \frac{n\pi x}{l} \cos \frac{n\pi c t}{l}$$

Most general solution :-

$$y(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \cos \frac{n\pi c t}{l}$$

Apply IC - 2

$$y_0 \sin^3 \frac{\pi x}{l} = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$y_0 \left(\frac{3}{4} \sin \frac{\pi x}{l} - \frac{1}{4} \sin \frac{3\pi x}{l} \right) = b_1 \sin \frac{\pi x}{l} + b_2 \sin \frac{2\pi x}{l} + b_3 \sin \frac{3\pi x}{l}$$

On Comparison,

$$b_1 = \frac{3y_0}{4}, \quad b_2 = 0, \quad b_3 = -\frac{y_0}{4}$$

$$\boxed{y(x,t) = \frac{3y_0}{4} \sin \frac{\pi x}{l} \cos \frac{\pi c t}{l} - \frac{y_0}{4} \sin \frac{3\pi x}{l} \cos \frac{3\pi c t}{l}}$$

Ans

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Q.1. Show how the wave equation $c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$ Can be solved by the method of separation of variables. If the initial displacement and velocity of a string stretched between $x = 0$ and $x = l$ are given by $y = f(x)$ and $\frac{\partial y}{\partial t} = g(x)$, determine the constants in the series solution. . (UPTU 2014)

Q.2 A tightly stretched flexible string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $F(x) = \mu x(l - x)$, μ is a constant and then released. Find the displacement $y(x, t)$ of any point x of the string at any time $t > 0$. (M.T.U. 2015)

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DPP-5 Maths-IV Unit-2

Topic- Solution of one dimmensional Wave Equation

Q-1: Show that the wave equation $c^2 \frac{\delta^2 y}{\delta x^2} = \frac{\delta^2 y}{\delta t^2}$ can be solved by the method of separation of variables. If the initial displacement and velocity of a string stretched $x=0$ and $x=l$ are given by $y=f(x)$ and $\frac{\delta y}{\delta t} = g(x)$, determine constants in series solution?

The equation of string is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

Length of string $(l) = l$

Boundry conditions, At $x=0$, $y=0$ BC-1
At $x=l$, $y=0$ BC-2

Initial conditions, At $t=0$, $y=f(x)$ IC-1
At $t=0$, $\frac{\delta y}{\delta t} = g(x)$ IC-2

Most suitable solution,

$$y = (C_1 \cos px + C_2 \sin px)(C_3 \cos cpt + C_4 \sin cpt)$$

pply Boundry condition-1,

$$0 = C_1 (C_3 \cos cpt + C_4 \sin cpt)$$

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$\boxed{C_1 = 0}$

Updated solution :

$$y = C_2 \sin px \, (C_3 \cos cpt + C_4 \sin cpt)$$

Apply BC-2 ;

$$0 = C_2 \sin pl \, (C_3 \cos cpt + C_4 \sin cpt)$$

$$\sin pl = 0$$
$$pl = n\pi$$
$$p = \frac{n\pi}{l}$$

Updated solution :

$$y = C_2 \sin \frac{n\pi}{l} x \left(C_3 \cos c \frac{n\pi}{l} t + C_4 \sin \frac{n\pi c}{l} t \right)$$

Apply IC-1 ;

$$f(x) = C_2 \sin \frac{n\pi}{l} x \cdot C_3$$

$$f(x) = C_2 C_3 \sin \frac{n\pi x}{l}$$

$$f(x) = \sum_{1}^{\infty} a_n \sin \frac{n\pi x}{l}$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cdot \sin \frac{n\pi x}{l} \, dx$$

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Apply IC-2;

$$\therefore a_n = C_2 C_3$$
$$\therefore b_n = C_2 C_4$$

$$\frac{\partial y}{\partial t} = \frac{\pi c}{l} \sum_{n=1}^{\infty} \left(-n a_n \sin \frac{n\pi ct}{l} + n b_n \cos \frac{n\pi ct}{l} \right) \sin \frac{n\pi x}{l}$$

$$g(x) = n b_n \sin \frac{n\pi x}{l} \times \frac{\pi c}{l}$$

$$\frac{n\pi c}{l} b_n = \frac{2}{l} \int_0^l g(x) \cdot \sin \frac{n\pi x}{l} \, dx$$

$$b_n = \frac{2}{n\pi c} \int_0^l g(x) \cdot \sin \frac{n\pi x}{l} \, dx$$

Hence the required solution,

$$\boxed{y(x,t) = \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi ct}{l} + b_n \sin \frac{n\pi ct}{l} \right) \sin \frac{n\pi x}{l}}$$

where, $$a_n = \frac{2}{l} \int_0^l f(x) \cdot \sin \frac{n\pi x}{l} \, dx$$

$$b_n = \frac{2}{n\pi c} \int_0^l g(x) \cdot \sin \frac{n\pi x}{l} \, dx$$

Q-2: A tightly stretched flexible string has its ends fixed at $x=0$ and $x=l$. At time $t=0$, the string is given a shape defined by $F(x) = \mu x(l-x)$, μ is a constant and then released. Find the displacement $y(x,t)$ of any point x of the string at any time $t>0$.

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The equation of string is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2}$$

Boundry conditions;

At $x = 0$, $y = 0$ BC-1

At $x = l$, $y = 0$ BC-2

Initial conditions;

At $t = 0$, $\frac{\delta y}{\delta t} = 0$ IC-1

At $t = 0$, $y = \mu x(l-x)$ IC-2

Most suitable solution,

$$y = (C_1 \cos px + C_2 \sin px)(C_3 \cos cpt + C_4 \sin cpt)$$

Apply BC-1;

$$0 = C_1 (C_3 \cos cpt + C_4 \sin cpt)$$

$$\boxed{C_1 = 0}$$

Updated solution,

$$y = C_2 \sin px \ (C_3 \cos cpt + C_4 \sin cpt)$$

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Apply BC-2;

$$0 = C_2 \sin pl \left(C_3 \cos cpt + C_4 \sin cpt\right)$$

$$\sin pl = 0$$
$$pl = n\pi$$
$$p = \frac{n\pi}{l}$$

Updated solution:-

$$y = C_2 \sin \frac{n\pi}{l} x \left(C_3 \cos c\frac{n\pi}{l} t + C_4 \sin \frac{n\pi c t}{l}\right)$$

Apply IC-1;

$$\frac{\delta y}{\delta t} = \frac{n\pi c}{l}\left[-C_3 \sin \frac{n\pi c t}{l} + C_4 \cos \frac{n\pi c t}{l}\right] \cdot C_2 \sin \frac{n\pi x}{l}$$

$$t = 0, \quad \frac{\delta y}{\delta t} = 0$$

$$0 = \frac{n\pi c}{l} C_2 C_4 \sin \frac{n\pi x}{l}$$

$$\boxed{C_4 = 0}$$

Updated solution,

$$y(x,t) = C_3 C_2 \cos \frac{n\pi c t}{l} \sin \frac{n\pi x}{l}$$

Apply IC-2

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$$u(lx-x^2) = \sum_{1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

Using half range sine series

$$b_n = \frac{2}{l}\int_0^l u(lx-x^2)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2u}{l}\left[\left\{(lx-x^2)\left(\frac{-\cos\frac{n\pi x}{l}}{n\pi/l}\right)\right\}_0^l - \int_0^l (l-2x)\left(\frac{-\cos\frac{n\pi x}{l}}{n\pi/l}\right)dx\right]$$

$$= \frac{2u}{l}\left[\frac{l}{n\pi}\int_0^l (l-2x)\cos\frac{n\pi x}{l}\,dx\right]$$

$$= \frac{2u}{n\pi}\left[\left\{(l-2x)\frac{\sin\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right\}_0^l - \int_0^l (-2)\frac{\sin\frac{n\pi x}{l}}{\frac{n\pi}{l}}\,dx\right]$$

$$= \frac{2u}{n\pi}\cdot\frac{2l}{n\pi}\int_0^l \sin\frac{n\pi x}{l}\,dx$$

$$= \frac{4ul}{n^2\pi^2}\left(\frac{-\cos\frac{n\pi x}{l}}{n\pi/l}\right)_0^l$$

$$= \frac{4ul^2}{n^3\pi^3}\left(-\cos n\pi + 1\right)$$

$$= \frac{4ul^2}{n^3\pi^3}\left[1-(-1)^n\right]$$

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$$b_n = \frac{4ul^2}{n^3\pi^3}\left[1-(-1)^n\right]$$

$$y(x,t) = \frac{4ul^2}{\pi^3}\sum_{n=1}^{\infty}\left[\frac{1-(-1)^n}{n^3}\right]\cos\frac{n\pi ct}{l}\sin\frac{n\pi x}{l}$$

For even $\Rightarrow$ $y = 0$

For odd $\Rightarrow$ $y = \frac{8ul^2}{\pi^3}\sum_{n=1}\frac{1}{(2n-1)^3}\sin\frac{(2n-1)\pi x}{l}\cos\frac{(2n-1)\pi ct}{l}$

Ans

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DPP- 6

Topic : One Dimensional Heat Equation

Q.1. A *rod of length l with insulated sides is initially at a uniform temperature* u_0. *Its ends are suddenly cooled to 0°C and are kept* at *that temperature. Find the temperature function* $u(x, t)$. *(U.P.T.U. 2015)*

Q.2. Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ where the boundary conditions are $u(0, t) = 0, u(l, t) = 0\ (t > 0)$ and the initial condition $u(x, 0) = x$: l being the length of the bar. *(U.P.T.U. 2015)*

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DPP-6 Maths-IV Unit-2

Topic- One Dimensional Heat Equation

Q-1: A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled at 0°C and are kept at that temperature. Find the temperature function $u(x,t)$.

Sol:- Boundry conditions

At $x=0$, $u=0$ $\Rightarrow u(0,t)=0°C$ BC-1

At $x=l$, $u=0$ $\Rightarrow u(l,t)=0°C$ BC-2

$u_s(x) = a+bx$

Apply BC-1

$0 = a+0$
$a = 0$

$u_s(x) = bx$
$0 = bl$
$b = 0$

$$\boxed{u_s(x)=0}$$

$$u(x,t) = u_s(x) + u_t(x,t)$$

u_s: Steady State, u_t: Transient state

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$$u(x,t) = u_t(x,t)$$

$$\boxed{u(x,t) = (C_1 \cos px + C_2 \sin px)\ C_3 e^{-c^2p^2t}}$$

Apply BC-1

$$0 = C_1 C_3 e^{-c^2p^2t}$$

$$\boxed{C_1 = 0}$$

Updated solution,

$$u(x,t) = C_2 \sin px \times C_3 e^{-c^2p^2t}$$

Apply BC-2;

$$0 = C_2 \sin pl \times C_3 e^{-c^2p^2t}$$

$$\sin pl = 0$$
$$pl = n\pi$$
$$p = \frac{n\pi}{l}$$

Updated solution-

$$u(x,t) = C_2 \sin \frac{n\pi}{l} x \times C_3 e^{-c^2\frac{n^2\pi^2}{l^2}t}$$

$$u(x,t) = C_2C_3 \sin \frac{n\pi}{l} x \times e^{-c^2\frac{n^2\pi^2}{l^2}t}$$

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Most general solution,

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \times e^{-\frac{c^2 n^2 \pi^2 t}{l^2}}$$

Apply IC, $t = 0$

$$u_0 = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \qquad -(11)$$

Using Half range sine series,

$$b_n = \frac{2}{l}\int_0^l u_0 \sin\frac{n\pi x}{l}\, dx$$

$$b_n = \frac{2u_0}{l}\left[\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right]_0^l$$

$$b_n = \frac{2u_0}{\cancel{l}} \times \frac{\cancel{l}}{n\pi}\left[-\cos n\pi + 1\right]$$

$$b_n = \frac{2u_0}{n\pi}\left[1 - (-1)^n\right]$$

$$b_n = \begin{cases} 0 & \text{when } n \text{ is even} \\ \frac{4u_0}{n\pi} & \text{when } n \text{ is odd} \end{cases}$$

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Put b_n in (11):

$$u(x,t) = \frac{4u_0}{\pi} \sum_{n=1,3,5..}^{\infty} \frac{1}{n} \sin\frac{n\pi x}{l}\, e^{-\frac{c^2 n^2 \pi^2 t}{l^2}}$$

$$= \frac{4u_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\frac{(2n-1)\pi x}{l}\, e^{-\frac{c^2 (2n-1)^2 \pi^2 t}{l^2}}$$

Ans

Q-2: Determine the solution of one-dimensional heat equation $\frac{\delta u}{\delta t} = c^2 \frac{\delta^2 u}{\delta x^2}$ where boundry conditions are $u(0,t) = 0 = u(l,t)$ and the initial condition $u(x,0) = x$: l being the length of bar.

Sol:- $$\frac{\delta u}{\delta t} = c^2 \frac{\delta^2 u}{\delta x^2} \quad -(1)$$

length of rod = l m

Boundry conditions, At $x = 0$, $u = 0$ BC-1
At $x = l$, $u = 0$ BC-2

Initial condition, At $t = 0$, $u = f(x) = x$

$$u(x,t) = u_s(x) + u_t(x,t)$$

$$u_s(x) = 0$$

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$u(x,t) = u_t(x,t)$

Most suitable solution,

$$u(x,t) = (C_1 \cos px + C_2 \sin px)\, C_3 e^{-c^2p^2t}$$

Apply BC-1;

$$0 = C_1 C_3 e^{-c^2p^2t}$$

$$\boxed{C_1 = 0}$$

Updated solution $\Rightarrow$ $u(x,t) = C_2 \sin px \times C_3 e^{-c^2p^2t}$

Apply BC-2

$$0 = C_2 \sin pl \times C_3 e^{-c^2p^2t}$$

$$\sin pl = 0$$

$$pl = n\pi$$

$$\boxed{p = \frac{n\pi}{l}}$$

Updated solution $\Rightarrow$

$$u(x,t) = C_2 \sin \frac{n\pi}{l} x \times C_3 e^{-c^2p^2t}$$

Most General sol,

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \times e^{-c^2p^2t} \quad -(11)$$

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Apply IC :-

$$x = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

Using Half-range sine series

$$b_n = \frac{2}{l} \int_0^l x \times \sin \frac{n\pi x}{l} \, dx$$

I II

$$b_n = \frac{2}{l} \left[x \times \frac{-\cos \frac{n\pi x}{l}}{n\pi/l} - \int \frac{-\cos \frac{n\pi x}{l}}{n\pi/l} \right]_0^l$$

$$b_n = \frac{2}{l} \left[x \times \frac{-\cos \frac{n\pi x}{l}}{n\pi/l} + \frac{\sin \frac{n\pi x}{l}}{n^2\pi^2/l^2} \right]_0^l$$

$$b_n = \frac{2}{l} \left[\frac{l \times -\cos n\pi}{n\pi/l} + \frac{\sin n\pi}{n^2\pi^2/l^2} \right]$$ ($\sin n\pi = 0$)

$$b_n = \frac{2}{l} \times l \times \frac{-\cos n\pi}{n\pi/l}$$

$$b_n = \frac{2l}{n\pi} (-\cos n\pi)$$

Put b_n in eq (11)

$$u(x,t) = -\frac{2l}{\pi} \sum_{n=1}^{\infty} \frac{\cos n\pi}{n} \sin \frac{n\pi x}{l} \, e^{-\left(\frac{c^2 n^2 \pi^2 t}{l^2}\right)}$$

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Q.1. The temperature distribution in a bar of length π which is perfectly insulated at ends $x = 0$ and $x = \pi$ is governed by partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

Assuming the initial temperature distribution as $u(x, 0) = f(x) = \cos 2x$, find the temperature distribution at any instant of time. **(M.T.U. 2011)**

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DPP-7 Maths - IV Unit-2

Topic - One Dimensional Heat Equation

Q-1 : The temperature distribution in a bar of length π which is perfectly insulated at ends $x=0$ and $x=\pi$ is governed by partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Assuming the initial temperature distribution as $u(x,0) = f(x) = \cos 2x$, find the temperature distribution at any instant of time.

Sol :- Length of rod $(l) = \pi$

Boundry conditions

At $x = 0$, $\frac{\partial u}{\partial x} = 0$ BC-1

At $x = \pi$, $\frac{\partial u}{\partial x} = 0$ BC-2

Initial condition :

At $t = 0$, $u = \cos 2x$

$u(x,t) = u_s(x) + u_t(x,t)$

$u_s(x) = a + bx$

$\frac{\partial u}{\partial x} = bx$

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Apply BC-1

$b = 0$

$\boxed{u_s = a}$

$u(x,t) = a + u_t(x,t)$

Most suitable solution :

$u(x,t) = a + (C_1 \cos px + C_2 \sin px)\, C_3 e^{-p^2c^2t}$

$\frac{\delta u}{\delta x} = 0 + (-C_1 p \sin px + p C_2 \cos px)\, C_3 e^{-c^2p^2t}$

Apply BC-1

$0 = C_2 p \times C_3 e^{-c^2p^2t}$

$\boxed{C_2 = 0}$

Updated solution :

$u(x,t) = a + (C_1 \cos px)\, C_3 e^{-c^2p^2t}$

$\frac{\delta u}{\delta x} = 0 + (-C_1 p \sin px)\, C_3 e^{-c^2p^2t}$

Apply BC-2;

$0 = -C_1 p \sin p\pi \times C_3 e^{-p^2c^2t}$

$\sin p\pi = 0$

$p\pi = n\pi$

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$\boxed{p = n}$

Updated solution $\Rightarrow$ $u = a + (C_1 \cos nx)\, C_3\, e^{-c^2 n^2 t}$

Most general solution :-

$$u = a + \sum_{n=1}^{\infty} b_n \cos nx \times e^{-c^2 n^2 t} \quad - (11)$$

Apply IC

$$\cos 2x = a + b_1 \cos x + b_2 \cos 2x + \cdots\cdots$$

On comparison $\Rightarrow$ $a = 0$, $b_2 = 1$, $b_1 = 0$

From equation (11)

$$u(x,t) = a + \sum_{n=1}^{\infty} b_n \cos nx \times e^{-c^2 n^2 t}$$

$$u(x,t) = a + b_1 \cos x \times e^{-c^2 t} + b_2 \cos 2x \times e^{-c^2 4t} + \cdots\cdots$$

$$u(x,t) = 0 + 0 + \cos 2x \times e^{-4c^2 t}$$

$$u(x,t) = \cos 2x \times e^{-4c^2 t}$$

$$u(x,t) = e^{-4c^2 t} \times \cos 2x$$

* From given equation in question $\Rightarrow$ $c^2 = 1$

$$\boxed{u(x,t) = e^{-4t} \times \cos 2x}$$ Ans

Gateway Classes

DPP- 8

Topic : One Dimensional Heat Equation

Q.1. (i) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t. [G.B.T.U. (A.G.) 2011, U.K.T.U. 2011]

(ii) Find also the temperature if the change consists of raising the temperature of A to 20°C and reducing that of B to 80°C.

Gateway Classes

DPP-8 Maths-IV Unit-2

Topic- One Dimensional Heat Equation

Q-1 (i) An insulated rod length l has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t.

ii) Find also the temperature if the change consists of raising of the temperature of A to 20°C and reducing that of B to 80°C.

Sol-i) $$\frac{\delta u}{\delta t} = c^2 \frac{\delta^2 u}{\delta x^2} \quad -(1)$$

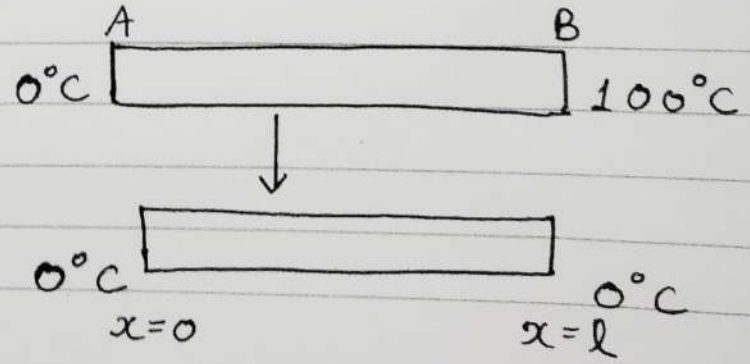
length of bar (l) = l cm

Boundry conditions,

At $x = 0$, $u = 0°$ BC-1
At $x = l$, $u = 0°$ BC-2

Initial condition,

$$u = T_A + \left(\frac{T_B - T_A}{l}\right) x$$



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$$u = 0 + \frac{100x}{l}$$

At $t = 0$, $u = \frac{100x}{l}$ — IC

Complete solution of (1)

$$u(x,t) = u_s(x) + u_s(x,t) \quad — (8)$$

Steady state solution -

$$u_s(x) = 0 + 0$$
$$= 0$$

$$\boxed{u_s(x) = 0}$$

$$u(x,t) = u_s(x,t)$$

$$u(x,t) = (C_1 \cos px + C_2 \sin px) C_3 e^{-c^2p^2t}$$

Apply BC-1;

$$0 = C_1 C_3 e^{-c^2p^2t}$$

$$\boxed{C_1 = 0}$$

Updated solution,

$$u(x,t) = C_2 \sin px \times C_3 e^{-c^2p^2t}$$

Apply BC-2

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$$0 = C_2 \sin pl \times C_3 e^{-c^2p^2t}$$

$$\sin pl = 0$$

$$pl = n\pi$$

$$\boxed{p = \frac{n\pi}{l}}$$

Updated solution : $u(x,t) = C_2 \sin \frac{n\pi}{l} x \times C_3 e^{-c^2 \frac{n^2\pi^2}{l^2} t}$

Most general solution :

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x \times e^{-c^2 \frac{n^2\pi^2}{l^2} t} \quad \text{--- (12)}$$

Apply Initial condition -

$$\frac{100x}{l} = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x$$

Apply Half range sine series,

$$b_n = \frac{2}{l} \int_0^l \underbrace{\frac{100x}{l}}_{I} \underbrace{\sin \frac{n\pi}{l} x}_{II} \, dx$$

$$b_n = \frac{200}{l^2} \left[\left\{ x \left(\frac{-\cos \frac{n\pi x}{l}}{n\pi/l} \right) \right\}_0^l - \int_0^l 1 \cdot \left(\frac{-\cos \frac{n\pi x}{l}}{n\pi/l} \right) dx \right]$$

$$b_n = \frac{200}{l^2} \left[\frac{-l^2}{n\pi} \cos n\pi + \frac{l}{n\pi} \left(\frac{\sin \frac{n\pi x}{l}}{n\pi/l} \right)_0^l \right]$$

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$$b_n = \frac{200}{l^2}\left[\frac{-l^2}{n\pi}\cos n\pi + \frac{l^2}{n^2\pi^2}(\sin n\pi - \sin 0)\right]$$

$$b_n = \frac{200}{l^2} \times \frac{-l^2}{n\pi}\cos n\pi$$

$$b_n = \frac{-200}{n\pi}(-1)^n$$

Put value of b_n in (12)

$$u(x,t) = \frac{-200}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{l}\, e^{-\frac{c^2n^2\pi^2 t}{l^2}}$$

Answer

ii) Boundry conditions,

At $x = 0$, $u = 20^\circ C$ BC-1

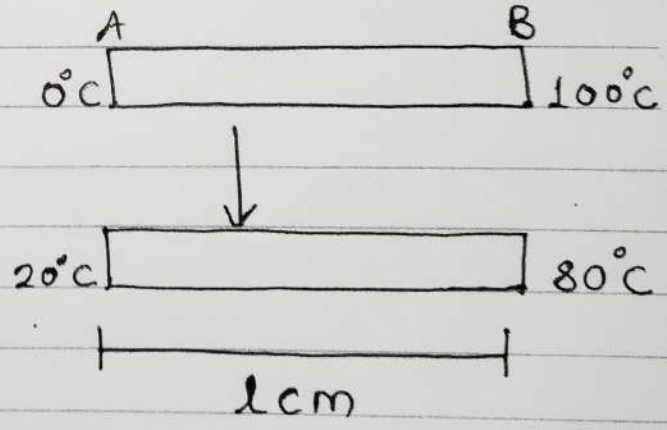
At $x = l$, $u = 80^\circ C$ BC-2

Initial condition

$$u = T_A + \left(\frac{T_B - T_A}{l}\right)x$$

$$u = 0 + \frac{100x}{l}$$

$$u = \frac{100x}{l}$$



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Complete solution of (1)

$$u(x,t) = u_s(x) + u_s(x,t)$$

$$u(x,t) = a + bx + (C_1 \cos px + C_2 \sin px) C_3 e^{-c^2p^2t} \quad -(8)$$

Steady state solution,

$$u_s(x) = T_A + \left(\frac{T_B - T_A}{l}\right) x$$

$$= 20 + \left(\frac{80-20}{l}\right) x$$

$$\boxed{u_s(x) = 20 + \frac{60x}{l}}$$

Put in eq -(8)

$$u(x,t) = 20 + \frac{60x}{l} + (C_1 \cos px + C_2 \sin px) C_3 e^{-c^2p^2t} \quad -(9)$$

Apply BC-1;

$$20 = 20 + C_1 C_3 e^{-c^2p^2t}$$

$$0 = C_1 C_3 e^{-c^2p^2t}$$

$$\boxed{C_1 = 0}$$

Updated solution ⇒

$$u(x,t) = 20 + \frac{60x}{l} + C_2 \sin px \times C_3 e^{-c^2p^2t} \quad -(10)$$

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Apply BC-2 ;

$$80 = 20 + 60 + C_2 \sin pl \times C_3 e^{-c^2 p^2 t}$$

$$80 - 80 = C_2 \sin pl \times C_3 e^{-c^2 p^2 t}$$

$$\sin pl = 0$$

$$pl = n\pi$$

$$\boxed{p = \frac{n\pi}{l}}$$

Updated solution :-

$$u(x,t) = 20 + \frac{60x}{l} + C_2 \sin \frac{n\pi}{l} x \times C_3 e^{-c^2 \frac{n^2 \pi^2}{l^2} t} \quad -(11)$$

Most general solution,

$$u(x,t) = 20 + \frac{60x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \times e^{-c^2 \frac{n^2 \pi^2}{l^2} t} \quad -(12)$$

Apply Initial conditions,

$$\frac{100x}{l} = 20 + \frac{60x}{l} \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \times 1$$

$$\frac{40x}{l} - 20 = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

Apply Half range sine series,

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$$b_n = \frac{2}{l}\int_0^l \left(\frac{40x}{l} - 20\right) \sin\frac{n\pi x}{l}\, dx$$

I II

$$b_n = \frac{2}{l}\left[\left\{\left(\frac{40x}{l} - 20\right)\left(\frac{-\cos\frac{n\pi x}{l}}{n\pi/l}\right)\right\}_0^l - \int_0^l \frac{40}{l}\cdot\left(\frac{-\cos\frac{n\pi x}{l}}{n\pi/l}\right)dx\right]$$

$$b_n = \frac{2}{l}\left[\frac{-20l}{n\pi}\cos n\pi - \frac{20l}{n\pi} + \frac{40}{n\pi}\left(\frac{\sin\frac{n\pi x}{l}}{n\pi/l}\right)_0^l\right]$$

$$b_n = \frac{2}{l}\left[\frac{-20l}{n\pi}\cos n\pi - \frac{20l}{n\pi} + \frac{40l}{n^2\pi^2}\left(\sin n\pi - \sin 0\right)\right]$$

$$b_n = -\frac{40}{n\pi}\cos n\pi - \frac{40}{n\pi}$$

$$b_n = -\frac{40}{n\pi}\left(1 + \cos n\pi\right)$$

$$b_n = -\frac{40}{n\pi}\left(1 + (-1)^n\right)$$

→ 0, when n is odd

→ $-\frac{80}{n\pi}$, when n is even

$$u(x,t) = 20 + \frac{60x}{l} - \frac{80}{\pi}\sum_{\substack{n=2,4,..n \\ (\text{even})}}^{\infty} \frac{1}{n}\sin\frac{n\pi x}{l}\, e^{-\left(\frac{n\pi c}{l}\right)^2 t}$$

$$u(x,t) = 20 + \frac{60x}{l} - \frac{40}{\pi}\sum_{n=1}^{\infty} \frac{1}{m}\sin\frac{2m\pi x}{l}\, e^{-\frac{4c^2m^2\pi^2 t}{l^2}}$$

{For even
Takes $n = 2m$

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Q.1. Solve temperature distribution in a bar of length π which is perfectly insulated at ends $x = 0$ and $x = \pi$ is governed by partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 < x < \pi, 0 < y < \pi,$$ **which satisfies the conditions :**

$$u(0,y) = u(\pi, y) = u(x, \pi) = 0 \text{ and } u(x, 0) = sin^2 x$$ (U.K.T.U. 2011)

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QPP-9 Maths - IV Unit - 2

Topic- Two Dimensional Heat Equation (Laplace Equation)

Q-1: Solve temperature distribution in a bar of length π which is perfectly insulated at ends $x=0$ and $x=\pi$ which is governed by partial differential equation $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0$, $0 < x < \pi$, $0 < y < \pi$, which satisfies the conditions:

$u(0,y) = u(\pi,y) = u(x,\pi) = 0$ and $u(x,0) = \sin^2 x$

Sol- Laplace Equation

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0 \quad -(1)$$

where $u(x,y)$ is steady state temp of plate

Under the Boundry condition;

$u(0,y) = 0$ BC-1

$u(\pi,y) = 0$ BC-2

$u(x,\pi) = 0$ BC-3

$u(x,0) = \sin^2 x$ BC-4

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Solution of Laplace equation

$$u(x,y) = (C_1 + C_2 x)(C_3 + C_4 y) \quad -(5)$$

$$u(x,y) = (C_1 e^{px} + C_2 e^{-px})(C_3 \cos py + C_4 \sin py) \quad -(6)$$

$$u(x,y) = (C_1 \cos px + C_2 \sin px)(C_3 e^{py} + C_4 e^{-py}) \quad -(7)$$

Most suitable solution

$$u(x,y) = (C_1 \cos px + C_2 \sin px)(C_3 e^{py} + C_4 e^{-py})$$

Apply BC-1

$$0 = C_1 (C_3 e^{py} + C_4 e^{-py})$$

$$\boxed{C_1 = 0}$$

Updated solution, $u(x,y) = C_2 \sin px (C_3 e^{py} + C_4 e^{-py}) \quad -(9)$

Apply BC-2,

$$0 = C_2 \sin p\pi (C_3 e^{py} + C_4 e^{-py})$$

$$\sin p\pi = 0$$
$$p\pi = n\pi$$
$$\boxed{p = n}$$

Updated solution: $u(x,y) = C_2 \sin nx (C_3 e^{ny} + C_4 e^{-ny}) \quad -(10)$

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$$u(x,y) = \sin nx \left(C_2C_3 e^{ny} + C_2C_4 e^{-ny}\right)$$

Let $C_2C_3 = A$ $\qquad$ $C_2C_4 = B$

Apply BC-3 :

$$0 = \sin nx \left(Ae^{n\pi} + Be^{-n\pi}\right)$$

$$Ae^{n\pi} + Be^{-n\pi} = 0$$

$$Ae^{n\pi} = -Be^{-n\pi} = -\frac{1}{2}B_n \text{ (say)}$$

$$\boxed{A = -\frac{B_n e^{-n\pi}}{2}} \qquad \boxed{B = \frac{B_n e^{+n\pi}}{2}}$$

Updated solution :

$$u(x,y) = \frac{1}{2}\sin nx \left(-B_n e^{-n\pi} e^{ny} + B_n e^{+n\pi} e^{-ny}\right)$$

$$u(x,y) = 1 \sin nx\, B_n \left[\frac{e^{n(\pi-y)} - e^{-n(\pi-y)}}{2}\right]$$

$$u(x,y) = \sin nx\, B_n \sinh n(\pi-y)$$

$$\because \boxed{\sinh x = \frac{e^x - e^{-x}}{2}}$$

Most general solution :

$$\boxed{u(x,y) = \sum_{n=1}^{\infty} B_n \sinh n(\pi-y) \sin nx} \quad -(11)$$

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Apply BC-4,

$$\sin^2 x = \sum_{n=1}^{\infty} B_n \sinh n\pi \sin nx$$

$$B_n \sinh n\pi = \frac{2}{\pi}\int_0^{\pi} \sin^2 x \sin nx\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} 2\sin^2 x \sin nx\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} (1-\cos 2x)\sin nx\, dx$$

$\cos 2x = 1 - 2\sin^2 x$
$2\sin^2 x = 1 - \cos 2x$

$$= \frac{1}{\pi}\int_0^{\pi}\left[\sin nx - \frac{2}{2}\cos 2x \sin nx\right] dx$$

$$= \frac{1}{\pi}\int_0^{\pi}\left[\sin nx - \frac{1}{2}\{\sin(n+2)x + \sin(n-2)x\{\right] dx$$

$\therefore 2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$$= \frac{1}{\pi}\left[\frac{-\cos nx}{n} + \frac{\cos(n+2)x}{2(n+2)} + \frac{\cos(n-2)x}{2(n-2)}\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{-2}{n} + \frac{1}{n+2} + \frac{1}{n-2}\{(-1)^n - 1\{\right], \text{ when } n \neq 2$$

$$B_n \sinh n\pi = \frac{1}{2\pi}\left[\frac{8}{n(n^2-4)}\{(-1)^n - 1\{\right], \text{ when } n \neq 2$$

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$$B_n \sinh n\pi = \begin{cases} \dfrac{-8}{\pi n(n^2-4)}, & \text{when } n \text{ is odd} \\ 0, & \text{when } n \text{ is even} \end{cases} \qquad n \neq 2$$

For $n = 2$,

$$B_2 \sinh 2\pi = \frac{2}{\pi} \int_0^{\pi} \sin^2 x \sin 2x \, dx$$

$\sin^2 x = 1 - \cos 2x$

$$= \frac{1}{\pi} \int_0^{\pi} \left(\sin 2x - \frac{1}{2} \sin 4x \right) dx$$

$$= \frac{1}{\pi} \left(-\frac{\cos 2x}{2} + \frac{1}{8} \cos 4x \right)_0^{\pi}$$

$$= 0$$

$$\boxed{B_2 = 0}$$

From the eq 11 :-

$$u(x,y) = \frac{-8}{\pi} \sum_{n=1,3,5..}^{\infty} \frac{\sin nx \sinh n(\pi - y)}{n(n^2-4) \sinh n\pi}$$

or

$$u(x,y) = \frac{-8}{\pi} \sum_{n=1}^{\infty} \frac{\sin (2m-1)x \sinh (2m-1)(\pi - y)}{(2m-1)\{(2m-1)^2 - 4\} \sinh (2m-1)\pi}$$

↓ For odd we put $\boxed{n = 2m-1}$

Ans

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Engineering Mathematics by Gulshan Sir

DPP- 10

Topic : Two Dimensional Heat Equation (Laplace Equation)

Part-2

Q.1. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy – plane with $u(x, 0) = 0$, $u(x, b) = 0$, $u(0,y) = 0$ and $u(a, y) = f(y)$ parallel to y-axis. (U.K.T.U. 2016)

Q.2 Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = 0$, $u(a, y) = 0$, $u(x, 0) = 0$ and $u(x, b) = x$. *[U.P.T.U. 2015]*

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DPP-10 Maths-IV Unit-2

Topic - Two Dimensional Heat Equation

Q-1: Solve the Laplace equation $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0$ in a rectangle in the xy-plane with $u(x,0) = 0$, $u(x,b) = 0$, $u(0,y) = 0$ and $u(a,y) = f(y)$ parallel to y-axis.

Sol- Let $u(x,y)$ be the steady state temp given by Laplace equation

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0 \quad -(1)$$

Using the Boundry conditions

$u(x,0) = 0$ BC-1

$u(x,b) = 0$ BC-2

$u(0,y) = 0$ BC-3

$u(a,y) = f(y)$ BC-4

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Most suitable solution of eq ①

When $k = p^2$

$$u(x,y) = \left(C_1 e^{px} + C_2 e^{-px}\right)\left(C_3 \cos py + C_4 \sin py\right)$$

Apply BC-1,

$$0 = \left(C_1 e^{px} + C_2 e^{-px}\right) \times C_3$$

$$\boxed{C_3 = 0}$$

Updated solution,

$$u(x,y) = \left(C_1 e^{px} + C_2 e^{-px}\right) \times C_4 \sin py$$

Apply BC-2;

$$0 = \left(C_1 e^{px} + C_2 e^{-px}\right) \times C_4 \sin pb$$

$$\sin pb = 0$$

$$pb = n\pi$$

$$\boxed{p = \frac{n\pi}{b}}$$

Updated solution,

$$u(x,y) = \left(C_1 e^{px} + C_2 e^{-px}\right) \times C_4 \sin \frac{n\pi}{b} y$$

$$u(x,y) = \left(C_1 e^{\frac{n\pi x}{b}} + C_2 e^{-\frac{n\pi x}{b}}\right) \times C_4 \sin \frac{n\pi}{b} y$$

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Apply BC-3,

$$0 = (C_1 + C_2) \times C_4 \sin \frac{n\pi}{b} y$$

$$C_1 + C_2 = 0$$

$$\boxed{C_1 = -C_2}$$

Updated solution;

$$u(x,y) = 2C_1C_4 \sin \frac{n\pi}{b} y \left(\frac{e^{\frac{n\pi x}{b}} - e^{-\frac{n\pi x}{b}}}{2} \right)$$

$$u(x,y) = 2C_1C_4 \sin \frac{n\pi}{b} y \sinh \frac{n\pi x}{b}$$

Most general solution :-

$$u(x,y) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{b} y \sinh \frac{n\pi x}{b}$$

Apply BC-4,

$$f(y) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{b} y \sinh \frac{n\pi a}{b}$$

Apply half range sine formula

$$\left(\sinh \frac{n\pi a}{b}\right) b_n = \frac{2}{b} \int_0^b f(y) \sin \frac{n\pi}{b} y \, dy$$

$$b_n = \frac{2}{b\left(\sinh \frac{n\pi a}{b}\right)} \int_0^b f(y) \sin \frac{n\pi}{b} y \, dy$$

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$$u(x,y) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi y}{b} \sinh\frac{n\pi x}{b}$$

where $b_n = \frac{2}{b\sinh\left(\frac{n\pi a}{b}\right)} \int_0^b f(y) \sin\frac{n\pi y}{b}\, dy$

Q-2 :- Solve $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0$ subject to the conditions

$u(0,y)$, $u(a,y) = 0$, $u(x,0) = 0$ and $u(x,b) = x$

Sol :- Let $u(x,y)$ be the steady state temp given by Laplace equation

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0 \quad -(1)$$

Using the boundry conditions,

$u(0,y) = 0$ BC-1
$u(a,y) = 0$ BC-2
$u(x,0) = 0$ BC-3
$u(x,b) = 0$ BC-4

Most suitable solution :-

$$u(x,y) = (C_1 \cos px + C_2 \sin px)\left(C_3 e^{py} + C_4 e^{-py}\right)$$

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Apply BC-1,

$$0 = C_1 (C_3 e^{py} + C_4 e^{-py})$$

$$\boxed{C_1 = 0}$$

Updated solution,

$$u(x,y) = C_2 \sin px \, (C_3 e^{py} + C_4 e^{-py})$$

Apply BC-2,

$$0 = C_2 \sin pa \, (C_3 e^{py} + C_4 e^{-py})$$

$$\sin pa = 0$$

$$pa = n\pi$$

$$\boxed{p = \frac{n\pi}{a}}$$

Updated solution,

$$u(x,y) = C_2 \sin \frac{n\pi x}{a} \left(C_3 e^{\frac{n\pi y}{a}} + C_4 e^{-\frac{n\pi y}{a}}\right)$$

Apply BC-3,

$$0 = C_2 \sin \frac{n\pi x}{a} (C_3 + C_4)$$

$$C_3 + C_4 = 0$$

$$\boxed{C_3 = -C_4}$$

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Updated solution,

$$u(x,y) = C_2 C_3 \sin \frac{n\pi x}{a} \left(e^{\frac{n\pi y}{a}} - e^{-\frac{n\pi y}{a}} \right)$$

Most general solution

$$u(x,y) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{a} \left(e^{\frac{n\pi y}{a}} - e^{-\frac{n\pi y}{a}} \right)$$

Apply BC-4,

$$x = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{a} \left(e^{\frac{n\pi b}{a}} - e^{-\frac{n\pi b}{a}} \right)$$

$$x = \sum_{n=1}^{\infty} b_n' \sin \frac{n\pi x}{a}$$

Apply Half range sine series

$$b_n' = \frac{2}{a} \int_0^a x \sin \frac{n\pi x}{a} \, dx$$

(I) (II)

$$b_n' = \frac{2}{a} \left[\left(-x \frac{\cos \frac{n\pi x}{a}}{n\pi/a} \right)_0^a - \int_0^a \frac{-\cos \frac{n\pi x}{a}}{n\pi/a} \right]$$

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$$b_n' = \frac{2}{a}\left[\left(\frac{-x\cos\frac{n\pi x}{a}}{n\pi/a}\right)_0^a + \left(\frac{\sin\frac{n\pi x}{a}}{\frac{n^2\pi^2}{a^2}}\right)_0^a\right]$$

$$b_n' = \frac{2}{a}\left[\left(\frac{a\cos n\pi - 0}{n\pi/a}\right) + \left(\frac{\sin n\pi - 0}{\frac{n^2\pi^2}{a^2}}\right)\right]_0^a$$

$$b_n' = \frac{2}{a} \times \frac{-a^2\cos n\pi}{n\pi}$$

$$b_n' = -\frac{2a}{\pi}\frac{\cos n\pi}{n}$$

$$b_n\left(e^{\frac{n\pi b}{a}} - e^{\frac{-n\pi b}{a}}\right) = -\frac{2a}{\pi}\frac{\cos n\pi}{n}$$

$$2b_n\left(\frac{e^{\frac{n\pi b}{a}} - e^{\frac{-n\pi b}{a}}}{2}\right) = -\frac{2a}{\pi}\frac{\cos n\pi}{n}$$

$$2b_n \sinh\left(\frac{n\pi b}{a}\right) = -\frac{2a}{\pi}\frac{\cos n\pi}{n}$$

$$b_n = \frac{-\frac{2a}{\pi}\frac{\cos n\pi}{n}}{2\sinh\left(\frac{n\pi b}{a}\right)}$$

Put value of b_n in most general solution

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$$u(x,y) = \sum_{n=1}^{\infty} \frac{-\frac{2a}{n\pi}\cos n\pi}{\sinh\left(\frac{n\pi b}{a}\right)} \left(\frac{e^{\frac{n\pi y}{a}} - e^{-\frac{n\pi y}{a}}}{2}\right) \sin\frac{n\pi x}{a}$$

$$u(x,y) = \frac{-2a}{\pi} \sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sinh\left(\frac{n\pi b}{a}\right)} \sinh\left(\frac{n\pi y}{a}\right) \sin\left(\frac{n\pi x}{a}\right)$$

$$u(x,y) = \frac{-2a}{\pi} \sum_{n=1}^{\infty} \frac{\cos n\pi \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi y}{a}\right)}{n\sinh\left(\frac{n\pi b}{a}\right)}$$

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DPP- 11

Topic : *Complex Fourier transform*

Q.1. *Find the Fourier Transform of* $F(x) = \begin{cases} x^2, & if\ |x| < a \\ 0, & if\ |x| \geq a \end{cases}$

Q.2. *Find the Fourier Transform of* $f(x) = e^{-|x|}$

Q.3. *Find the Fourier transform of* $F(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. *hence avaluate* $\int_0^\infty \frac{\sin x}{x} dx$. *(GBTU-2011)*

Q. 4. *Find the Fourier transform of Block function* $f(t)$ *of height 1 and duration a defined by* *(AKTU-2016)*

$$f(t) = \begin{cases} 1, & for\ |t| \leq \frac{a}{2} \\ 0, & otherwise \end{cases}$$

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DPP-11 Unit-2 Maths-IV

Topic - Complex Fourier Transform

Q-1: Find the Fourier Transform of $F(x) = \begin{cases} x^2, & \text{if } |x| < a \\ 0 & \text{if } F(x) > a \end{cases}$

Sol: $F(x) = \begin{cases} x^2 & -a < x < a \\ 0 & x > a \text{ or } x < -a \end{cases}$

By Fourier Transform

$$f(p) = \int_{-\infty}^{\infty} F(x)\, e^{ipx}\, dx$$

$$f(p) = \int_{-\infty}^{-a} F(x) e^{ipx} dx + \int_{-a}^{a} F(x) e^{ipx} dx + \int_{a}^{\infty} F(x) e^{ipx} dx$$

$$f(p) = 0 + \int_{-a}^{a} x^2 e^{ipx} dx + 0$$

$$f(p) = \int_{-a}^{a} \underset{I}{x^2} \underset{II}{e^{ipx}}\, dx$$

$$f(p) = \left[x^2 \left(\frac{e^{ipx}}{ip} \right) - \int \underset{I}{2x} \times \frac{\underset{II}{e^{ipx}}}{ip} \right]$$

$$f(p) = \left[x^2 \left(\frac{e^{ipx}}{p} \right) - 2 \left[x \times \frac{e^{ipx}}{i^2 p^2} - \left(\frac{e^{ipx}}{i^3 p^3} \right) \right] \right]_{-a}^{a}$$

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$$f(p) = \left[x^2 \left(\frac{e^{ipx}}{ip}\right) - 2x \left(\frac{e^{ipx}}{i^2p^2}\right) + 2 \left(\frac{e^{ipx}}{i^3p^3}\right) \right]_{-a}^{a}$$

$$f(p) = \left[a^2 \left(\frac{e^{ipa}}{ip}\right) - 2a \left(\frac{e^{ipa}}{i^2p^2}\right) + 2 \left(\frac{e^{ipa}}{i^3p^3}\right) - a^2 \left(\frac{e^{-ipa}}{ip}\right) - 2a \left(\frac{e^{-ipa}}{i^2p^2}\right) - 2 \left(\frac{e^{-ipa}}{i^3p^3}\right) \right]$$

$$f(p) = \frac{a^2}{ip} \left(e^{ipa} - e^{-ipa}\right) - \frac{2a}{i^2p^2} \left(e^{ipa} + e^{-ipa}\right) + \frac{2}{i^3p^3} \left(e^{ipa} - e^{-ipa}\right)$$

$$f(p) = \frac{2a^2}{p} \left(\frac{e^{ipa} - e^{-ipa}}{2i}\right) + \frac{4a}{p^2} \left(\frac{e^{ipa} + e^{-ipa}}{2}\right) - \frac{2 \times 2}{p^3} \left(\frac{e^{ipa} - e^{-ipa}}{2i}\right)$$

$$f(p) = \frac{2a^2}{p} \sin ap + \frac{4a}{p^2} \cos ap - \frac{4}{p^3} \sin ap$$

$$\boxed{f(p) = \left(\frac{2a^2}{p} - \frac{4}{p^3}\right) \sin ap + \frac{4a}{p^2} \cos ap}$$

Q-2: Find the fourier transform of $f(x) = e^{-|x|}$. Ans

$f(x) = e^{-|x|}$

By Fourier Transform

$$f(p) = \int_{-\infty}^{\infty} f(x) e^{ipx} \, dx$$

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$$f(p) = \int_{-\infty}^{0} f(x) e^{ipx} dx + \int_{0}^{\infty} f(x) e^{ipx} dx$$

$$f(p) = \int_{-\infty}^{0} e^{x} \times e^{ipx} dx + \int_{0}^{\infty} e^{-x} \times e^{ipx} dx$$

$$f(p) = \int_{-\infty}^{0} e^{(1+ip)x} dx + \int_{0}^{\infty} e^{(ip-1)x} dx$$

$$f(p) = \left[\frac{e^{(ip+1)x}}{1+ip}\right]_{-\infty}^{0} + \left[\frac{e^{-(1-ip)x}}{ip-1}\right]_{0}^{\infty}$$

$$f(p) = \left[\frac{e^{0} - e^{-\infty}}{1+ip}\right] + \left[\frac{e^{-\infty} - e^{0}}{ip-1}\right]$$

$$f(p) = \frac{1}{1+ip} + \frac{1}{1-ip}$$

$$f(p) = \frac{2}{1^2 - i^2 p^2}$$

$$\boxed{f(p) = \frac{2}{1+p^2}}$$

Q-3: Find the Fourier transform of $F(x) = \begin{cases} 1, & |x|<1 \\ 0, & |x|>1 \end{cases}$ hence evaluate $\int_{0}^{\infty} \frac{\sin x}{x} dx$

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Sol- $F(x) = \begin{cases} 1 & -1 < x < 1 \\ 0 & x > 1 \text{ or } x < -1 \end{cases}$

By Fourier Transform

$$f(p) = \int_{-\infty}^{\infty} F(x)\, e^{ipx}\, dx$$

$$f(p) = \int_{-\infty}^{-1} F(x) e^{ipx} + \int_{-1}^{1} F(x) e^{ipx} dx + \int_{1}^{\infty} F(x) e^{ipx} dx$$

$$f(p) = \int_{-1}^{1} e^{ipx} dx$$

$$f(p) = \left[\frac{e^{ipx}}{ip}\right]_{-1}^{1}$$

$$f(p) = \frac{1}{ip}\left[e^{ip} - e^{-ip}\right]$$

$$f(p) = \frac{2}{p}\left[\frac{e^{ip} - e^{-ip}}{2i}\right]$$

$$\boxed{f(p) = \frac{2\sin p}{p}}$$

By Inverse Fourier transform

$$F(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(p)\, e^{-ipx}\, dp$$

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$$F(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{p} \sin p \, e^{-ipx} \, dp$$

$$F(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{p} \sin p \, (\cos px - i \sin px) \, dp$$

$$F(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left(\underbrace{\frac{\sin p \cos px}{p}}_{\text{even}} - i \underbrace{\frac{\sin p \sin px}{p}}_{\text{odd}} \right) dp$$

By property of definite integral

$$F(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin p \cos px}{p} \, dp$$

$$\int_{-\infty}^{\infty} \frac{\sin p \cos px}{p} \, dp = \pi F(x)$$

Put $x = 0$

$$\int_{-\infty}^{\infty} \frac{\sin p}{p} \, dp = \pi F(0)$$

$$\int_{-\infty}^{\infty} \frac{\sin p}{p} \, dp = \pi$$

By property of Definite Integral

$$2 \int_{0}^{\infty} \frac{\sin p}{p} \, dp = \pi$$

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$$\int_0^\infty \frac{\sin p}{p}\, dp = \frac{\pi}{2}$$

By property of definite integral

$$\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$$

Q-4:- Find the Fourier transform of Block function $f(t)$ of height 1 and duration defined by -

$$f(t) = \begin{cases} 1, & \text{for } |t| \le a/2 \\ 0, & \text{otherwise} \end{cases}$$

Sol- $f(t) = \begin{cases} 1 & -\frac{a}{2} \le t \le \frac{a}{2} \\ 0 & t > \frac{a}{2} \text{ or } t < -\frac{a}{2} \end{cases}$

By Fourier Transform

$$f(p) = \int_{-\infty}^{\infty} f(t) e^{ipx}\, dx$$

$$f(p) = \int_{-\infty}^{-a/2} f(t) e^{ipx}\, dx + \int_{-\frac{a}{2}}^{a/2} f(t) e^{ipx}\, dx + \int_{a/2}^{\infty} f(t) e^{ipx}\, dx$$

$$F(p) = \int_{-a/2}^{a/2} e^{ipx}\, dx$$

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$$f(p) = \left[\frac{e^{ipx}}{ip}\right]_{-a/2}^{+a/2}$$

$$f(p) = \frac{1}{ip}\left[e^{ipa/2} - e^{-ipa/2}\right]$$

$$f(p) = \frac{2}{p}\left[\frac{e^{ipa/2} - e^{-ipa/2}}{2i}\right]$$

$$\boxed{f(p) = \frac{2}{p} \times \sin\frac{ap}{2}}$$

Ans

Gateway Classes

DPP- 12

Topic : *Fourier sine transform*

Q.1. *Find Fourier sine transform of* $\frac{1}{x}$

Q. 2. *Find the Fourier sine transform of Block function* $f(t)$ *of height 1 and duration a defined by*

$$f(t) = \begin{cases} sinx, & 0 < x < a \\ 0, & x > a \end{cases}$$

Q.3. *Find* $F_s^{-1}\left(\frac{e^{-ap}}{p}\right)$ *and hence evaluate* $F_s^{-1}\left(\frac{1}{p}\right)$. *(AKTU-2016)*

Gateway Classes

DPP-12 Mathr-IV Unit-2

Topic- Fourier Sine Transformation

Q-1:- Find Fourier sine transform of $\frac{1}{x}$.

Sol- By Fourier sine transform

$$f_s(p) = \int_0^\infty F(x) \sin px \, dx$$

$$f_s(p) = \int_0^\infty \frac{1}{x} \sin px \, dx$$

Let $\sin px = y$

$p\,dx = dy$

$$f_s(p) = \int_0^\infty \frac{1}{(y/p)} \times \sin y \times \frac{dy}{p}$$

$$f_s(p) = \int_0^\infty \frac{\sin y}{y}$$

We know $\int_0^\infty \frac{\sin y}{y} = \frac{\pi}{2}$

$$f_s(p) = \frac{\pi}{2}$$

Ans

Gateway Classes

Q-2:- Find the Fourier transform of Block Function $f(t)$ of height 1 and duration a defined by

$$f(t) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x > a \end{cases}$$

Sol:- By Fourier sine transform

$$f_s(p) = \int_0^a f(t) \sin px \, dx + \int_a^\infty f(t) \sin px \, dx$$

$$= \int_0^a \sin x \sin px \, dx + 0$$

$$= \frac{1}{2} \int_0^a 2 \sin x \sin px \, dx$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$= \frac{1}{2} \int_0^a \left(\cos(p-1)x - \cos(p+1)x \right) dx$$

$$= \frac{1}{2} \left[\frac{\sin(p-1)x}{p-1} - \frac{\sin(p+1)x}{p+1} \right]_0^a$$

$$f_s(p) = \frac{1}{2} \left[\frac{\sin(p-1)a}{p-1} - \frac{\sin(p+1)a}{p+1} \right]$$

Ans:-

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Q-3 - Find $F_s^{-1}\left(\frac{e^{-ap}}{p}\right)$ and hence evaluate $F_s^{-1}\left(\frac{1}{p}\right)$

Sol - By Inverse Fourier Sine Transform

$$F(x) = \frac{2}{\pi}\int_0^\infty F_s(p)\sin px\,dp$$

$$F(x) = \frac{2}{\pi}\int_0^\infty \frac{e^{-ap}}{p}\sin px\,dp = I \text{ (say)}$$

$$I = \frac{2}{\pi}\int_0^\infty \frac{e^{-ap}}{p}\sin px\,dp$$

differentiate wrt x

$$\frac{dI}{dx} = \frac{2}{\pi}\int_0^\infty \frac{e^{-ap}}{p}\frac{d}{dx}\sin px\,dp$$

$$\frac{dI}{dx} = \frac{2}{\pi}\int_0^\infty \frac{e^{-ap}}{\cancel{p}}\times\cancel{p}\times\cos px\,dp$$

$$\frac{dI}{dx} = \frac{2}{\pi}\left[\frac{e^{-ap}}{(-a)^2+x^2}(-a\cos px + x\sin px)\right]_0^\infty$$

$$= \frac{2}{\pi}\left[\frac{e^{-\infty}}{a^2+x^2}(-a\cos\infty + x\sin\infty) - \frac{e^0}{a^2+x^2}(-a\cos 0+\sin 0\right.$$

($e^{-\infty}\to 0$)

$$\boxed{\frac{dI}{dx} = \frac{2}{\pi}\times\frac{a}{a^2+x^2}}$$

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$$dI = \frac{2}{\pi} \times \frac{a}{a^2+x^2}\, dx$$

Integrate both side

$$I = \frac{2}{\pi} \int \frac{a}{a^2+x^2}\, dx$$

$$I = \frac{2}{\pi} \tan^{-1}\left(\frac{x}{a}\right) + C = F(x)$$

For $x = 0$

$$\boxed{C = 0}$$

$$\boxed{F(x) = F_S^{-1}\left(\frac{e^{-ap}}{p}\right) = \frac{2}{\pi} \tan^{-1}\left(\frac{x}{a}\right)}$$

Put $a = 0$

$$F_S^{-1}\left(\frac{1}{p}\right) = \frac{2}{\pi} \tan^{-1}\left(\frac{x}{0}\right)$$

$$F_S^{-1}\left(\frac{1}{p}\right) = \frac{2}{\pi} \tan^{-1}(\infty)$$

$$F_S^{-1}\left(\frac{1}{p}\right) = \frac{2}{\pi} \times \frac{\pi}{2}$$

$$\boxed{F_S^{-1}\left(\frac{1}{p}\right) = 1}$$

Ans

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DPP- 13

Topic : *Fourier cosine transform*

Q.1. *Find Fourier cosine transform* $F(x) = \begin{cases} x, & 0 < x < \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x < 1 \\ 0, & x > 1 \end{cases}$

Q. 2. *Find the Fourier cosine transform of* e^{-x^2}

Q.3. *Find Fourier cosine transform of* $F(x) = 2e^{-5x} + 2e^{-2x}$

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DPP-13 Maths-IV Unit-2

Q-1: Find Fourier cosine transform $F(x) = \begin{cases} x, & 0<x<1/2 \\ 1-x, & 1/2<x<1 \\ 0, & x>1 \end{cases}$

Sol- By Fourier cosine transform

$$f_c(p) = \int_0^\infty F(x)\cos px\,dx$$

$$f_c(p) = \int_0^{1/2} F(x)\cos px\,dx + \int_{1/2}^{1} F(x)\cos px\,dx + \int_1^\infty F(x)\cos px\,dx$$

$$f_c(p) = \int_0^{1/2} \underset{I}{x}\cos px\,dx + \int_{1/2}^{1} \underset{I}{(1-x)}\cos px\,dx + 0$$

$$f_c(p) = \left[x\left(\frac{\sin px}{p}\right) - 1\left(\frac{-\cos px}{p^2}\right)\right]_0^{1/2} + \left[(1-x)\left(\frac{\sin px}{p}\right) - (-1)\left(\frac{-\cos px}{p^2}\right)\right]_{1/2}^{1}$$

$$f_c(p) = \left[\frac{1}{2}\left(\frac{\sin p/2}{p}\right) + \frac{\cos p/2}{p^2} - \frac{1}{p^2}\right] + \left[-\frac{\cos p}{p^2} - \frac{1}{2}\left(\frac{\sin p/2}{p}\right) + \frac{\cos p/2}{p^2}\right]$$

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$$f_c(p) = \frac{1}{2}\left(\frac{\sin p/2}{p}\right) + \frac{\cos p/2}{p^2} - \frac{1}{p^2} - \frac{\cos p}{p^2} - \frac{1}{2}\left(\frac{\sin p/2}{p}\right) + \frac{\cos p/2}{p^2}$$

$$f_c(p) = \frac{2\cos p/2}{p^2} - \frac{\cos p}{p^2} - \frac{1}{p^2}$$

$$f_c(p) = \frac{2\cos p/2}{p^2} - \left(\frac{2\cos^2 p/2 - 1}{p^2}\right) - \frac{1}{p^2}$$

$$f_c(p) = \frac{2\cos p/2}{p^2} - \frac{2\cos^2 p/2}{p^2} + \frac{1}{p^2} - \frac{1}{p^2}$$

$$\boxed{f_c(p) = \frac{2\cos p/2\left(1 - \cos p/2\right)}{p^2}}$$

Ans

Q-2: Find the Fourier cosine transform of e^{-x^2}.

By Fourier cosine Transform,

$$f_c(p) = \int_0^\infty e^{-x^2}\cos px\,dx = I \text{ (say)} \quad -(1)$$

$$I = \int_0^\infty e^{-x^2}\cos px\,dx$$

Differentiate w.r.t 'p'

$$\frac{dI}{dp} = \int_0^\infty e^{-x^2}\frac{d}{dp}\cos px\,dx$$

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$$\frac{dI}{dp} = \int_0^\infty e^{-x^2} x(-x \sin px)\, dx$$

$$\frac{dI}{dp} = \int_0^\infty -x e^{-x^2} \sin px\, dx$$

(II, I)

$$\frac{dI}{dp} = \left[\sin px \cdot \int -x e^{-x^2} dx\right]_0^\infty - \int_0^\infty \left[p \cos px \cdot \int -x e^{-x^2} dx\right] dx$$

$\because \int -x e^{-x^2} dx \Rightarrow$ Let $-x^2 = t$

$-2x\,dx = dt$

$-x\,dx = \frac{dt}{2}$

$$= \int e^t \frac{dt}{2} = \frac{1}{2}\int e^t dt = \frac{e^t}{2} = \frac{e^{-x^2}}{2}$$

$$\frac{dI}{dp} = \left[\sin px \cdot \frac{e^{-x^2}}{2}\right]_0^\infty - p \int_0^\infty \cos px \cdot \frac{e^{-x^2}}{2} dx$$

$$\frac{dI}{dp} = [0 - 0] - \frac{p}{2} \underbrace{\int_0^\infty e^{-x^2} \cos px\, dx}_{I}$$

$$\frac{dI}{dp} = -\frac{p}{2} I$$

$$\frac{dI}{I} = -\frac{p}{2} dp$$

Integerate both side

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$$\log I = -\frac{p^2}{4} + \log C$$

$$\log I - \log C = -\frac{p^2}{4}$$

$$\frac{I}{C} = e^{-p^2/4}$$

$$\boxed{I = Ce^{-p^2/4}}$$

For $p = 0$

$$\boxed{I = C}$$

$$I_{p=0} = \int_0^\infty e^{-x^2} \cos 0 \, dx$$

$$= \int_0^\infty e^{-x^2} dx$$

We know

$$\boxed{\int_0^\infty e^{-x^2} dx = \sqrt{\frac{\pi}{2}}}$$

$$I = \sqrt{\frac{\pi}{2}} = C$$

$$\boxed{I = \sqrt{\frac{\pi}{2}}\, e^{-p^2/4}}$$

Ans

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Q-3: Find Fourier cosine transform of $F(x) = 2e^{-5x} + 2e^{-2x}$

$$F(x) = 2e^{-5x} + 2e^{-2x}$$

By Fourier cosine transform

$$f_c(p) = \int_0^\infty F(x) \cos px \, dx$$

$$f_c(p) = 2\int_0^\infty e^{-5x} \cos px \, dx + 2\int_0^\infty e^{-2x} \cos px \, dx$$

$$f_c(p) = 2\left[\frac{e^{-5x}}{(-5)^2+p^2}(-5\cos px + p\sin px)\right]_0^\infty + 2\left[\frac{e^{-2x}}{(-2)^2+p^2}(-2\cos px + p\sin px)\right]_0^\infty$$

$$f_c(p) = 2\left[0 - \frac{1}{25+p^2}(-5)\right] + 2\left[0 - \frac{1}{4+p^2}(-2)\right]$$

$$f_c(p) = \frac{10}{25+p^2} + \frac{4}{4+p^2}$$

$$\boxed{f_c(p) = \frac{10}{p^2+25} + \frac{4}{p^2+4}}$$

Ans

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