ENGINEERING MATHEMATICS

UNIT : Application of partial differential equations and Fourier Transform

Today's Target

- One Dimensional Heat Equation (Part – 2)
- PYQ
- DPP

Application of partial differential equations

1. Solution of one Dimensional Wave Equation
2. Solution of one Dimensional Heat Equation
3. Solution of two Dimensional heat Equation

One Dimensional Heat Equation :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

On the Basis of Boundary conditions, there are three types of question

Type -1: When both ends of rod are at 0^oC

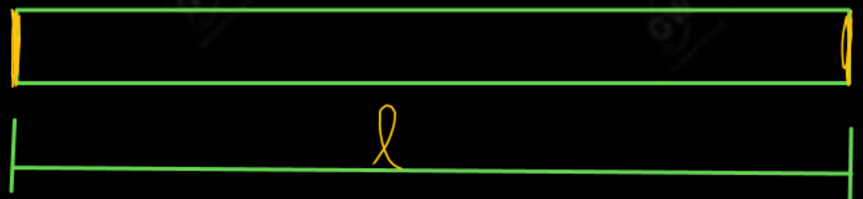
Type -2: When both ends of rod are perfectly insulated

Type -3: When both ends of rod are kept at different temperature.

Type -2 : When both ends of rod are perfectly insulated

Since ends of rod are insulated, no heat can transfer from either sides.

$\frac{\partial u}{\partial x}$



$$u(x,t) = u_s(x) + u_t(x,t)$$

Steady state | Transient state

Boundary Conditions

At $x = 0$, $\frac{\partial u}{\partial x} = 0$

At $x = l$, $\frac{\partial u}{\partial x} = 0$

$u_s = a + bx$

$\frac{\partial u}{\partial x} = 0 + b$

Apply BC-1

$0 = b$

$\boxed{u_s = a}$

$u(x,t) = a + u_t(x,t)$

$$\boxed{u(x,t) = a + (C_1 \cos px + C_2 \sin px) C_3 e^{-c^2 p^2 t}}$$

Q. 1:- Find the temperature distribution in a rod of length π which is totally insulated including the ends and the initial temperature distribution is $100 \cos x$. (U.P.T.U. 2015)

one-dimensional Heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{——} \quad (1)$$

length of rod $(l) = \pi$

Boundary conditions

At $x = 0$, $\frac{\partial u}{\partial x} = 0$ B.C-1

At $x = \pi$, $\frac{\partial u}{\partial x} = 0$ B.C-2

Initial condition

At $t = 0$, $u = 100 \cos x$ I.C

complete solution of (1)

$$u(x,t) = a + (c_1 \cos px + c_2 \sin px) c_3 e^{-c^2 p^2 t} \quad \text{——} \quad (2)$$

$$\frac{\partial u}{\partial x} = 0 + (-c_1 p \sin px + c_2 p \cos px) c_3 e^{-c^2 p^2 t}$$

Apply B.C-1

$$0 = c_2 p \times c_3 e^{-c^2 p^2 t}$$

$\Rightarrow C_2 = 0$

Updated solution

$$u(x,t) = a + (C_1 \cos px)\, C_3 e^{-c^2 p^2 t}$$ — (9)

$$\frac{\partial u}{\partial x} = 0 + (-C_1 p \sin px)\, C_3 e^{-c^2 p^2 t}$$

Apply BC-2

$$0 = -C_1 p \sin \pi p \times C_3 e^{-c^2 p^2 t}$$

$\Rightarrow \sin p\pi = 0$

$\sin p\pi = \sin n\pi$

$p\pi = n\pi$

$p = n$

Updated solution

$$u = a + (C_1 \cos nx)\, C_3 e^{-c^2 n^2 t}$$ — (10)

Most-General solution

$$u = a + \sum_{n=1}^{\infty} b_n \cos nx \times e^{-c^2 n^2 t}$$ — (11)

Apply I C

$$100\cos x = a + \sum_{n=1}^{\infty} b_n \cos nx$$

$$100\cos x = a + b_1 \cos x + b_2 \cos 2x + \dots$$

By comparison

$a = 0$

$b_1 = 100$

$b_2 = 0$

$b_3 = 0$

From (11)

$$u(x,t) = a + \sum_{n=1}^{\infty} b_n \cos nx \times e^{-c^2 n^2 t}$$

$$= a + b_1 \cos x \times e^{-c^2 t} + b_2 \cos 2x \times e^{-c^2 x^2 x t}$$

$$u(x,t) = 0 + 100 \cos x \times e^{-c^2 t}$$

$$\boxed{u(x,t) = 100 e^{-c^2 t} \cos x}$$

Q.1. The temperature distribution in a bar of length π which is perfectly insulated at ends $x = 0$ and $x = \pi$ is governed by partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

Assuming the initial temperature distribution as $u(x, 0) = f(x) = \cos 2x$, find the temperature distribution at any instant of time. **(M.T.U. 2011)**

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