DPP-3 Maths-IV Unit-3

Topic- Relation between μ_r and μ_r'

Q-1:- The first four moments of a distribution about $x=2$ are 1, 2·5, 5·5 and 16. Calculate first four moments about the mean and about origin.

Sol:- $A = 2$

$\mu_1' = 1$

$\mu_2' = 2.5$

$\mu_3' = 5.5$

$\mu_4' = 16$

Moments about mean $\boxed{\mu_1 = 0}$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2 = 2.5 - 1$

$\boxed{\mu_2 = 1.5}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3 = 5.5 - 3 \times 2.5 \times 1 + 2$

$\boxed{\mu_3 = 0}$

Gateway Classes

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 16 - 4 \times 5.5 \times 1 + 6 \times 2.5 \times 1 - 3$$

$$\mu_4 = 16 - 22 + 15 - 3$$

$$\boxed{\mu_4 = 6}$$

Moments about origin:-

$$\nu_1 = \mu_1' + A$$

$$\nu_1 = 1 + 2$$

$$\boxed{\nu_1 = 3} = \bar{x}$$

$$\nu_2 = \mu_2 + \bar{x}^2$$

$$\nu_2 = 1.5 + 9$$

$$\boxed{\nu_2 = 10.5}$$

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_3 = 0 + 3 \times 1.5 \times 3 + 27$$

$$\boxed{\nu_3 = 40.5}$$

Gateway Classes

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

$$\nu_4 = 6 + 4 \times 0 \times 3 + 6 \times 1.5 \times 9 + 81$$

$$\nu_4 = 6 + 81 + 81$$

$$\boxed{\nu_4 = 168}$$

Q-2:- In a certain distribution, the first four moments about the point $x = 4$ are $-1.5, 17, -30$ and 308. Find the moments about mean and about origin also calculate β_1 and β_2.

Sol:- $A = 4$

$\mu_1' = -1.5 \quad \mu_2' = 17 \quad \mu_3' = -30 \quad \mu_4' = 308$

Moments about mean $\Rightarrow$ $\boxed{\mu_1 = 0}$

$$\mu_2 = \mu_2' - \mu_1^2$$

$$\mu_2 = 17 - (-1.5)^2$$

$$\mu_2 = 17 - 2.25$$

$$\boxed{\mu_2 = 14.75}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_3 = -30 - 3 \times 17 \times -1.5 + 2 \times (-3.375)$$

Gateway Classes

$\mu_3 = -30 + 76.5 - 6.75$

$\boxed{\mu_3 = 39.75}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\mu_4 = 308 - 4 \times (-30) \times (-1.5) + 6 \times 17 \times (-1.5)^2 - 3 \times (-1.5)^4$

$\mu_4 = 308 - 180 + 229.5 - 15.18$

$\boxed{\mu_4 = 342.32}$

Moment about origin

$\nu_1 = \mu_1' + A$

$\nu_1 = -1.5 + 4$

$\boxed{\nu_1 = 2.5 = \bar{x}}$

$\nu_2 = \mu_2 + \bar{x}^2$

$\nu_2 = 14.75 + (2.5)^2$

$\boxed{\nu_2 = 21}$

Gateway Classes

$$\nu_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$$

$$\nu_3 = 39.75 + 3 \times 14.75 \times 2.5 + (2.5)^3$$

$$\boxed{\nu_3 = 165.995}$$

$$\nu_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$$

$$\nu_4 = 342.32 + 397.5 + 553.125 + 39.062$$

$$\boxed{\nu_4 = 1332}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(39.75)^2}{(14.75)^3} = \frac{1580.062}{3209.04}$$

$$\boxed{\beta_1 = 0.4923}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{342.32}{217.56} = 1.57$$

Q- The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50. Calculate mean, variance, μ_3 and μ_4.

Gateway Classes

Sol:- $A = 5$

$\mu_1' = 2, \quad \mu_2' = 20, \quad \mu_3' = 40, \quad \mu_4' = 50$

$\bar{x} = \mu_1' + A$

$\bar{x} = 2 + 5$

$\boxed{\bar{x} = 7}$

$\mu_2 = \mu_2' - \mu_1'^2$

$\mu_2 = 20 - 4$

$\boxed{\mu_2 = 16 = \text{Variance}}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3 = 40 - 3 \times 20 \times 2 + 2 \times 8$

$\mu_3 = 40 - 120 + 16$

$\boxed{\mu_3 = -64}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\mu_4 = 50 - 4 \times 40 \times 2 + 6 \times 20 \times 4 - 3 \times 16$

$\boxed{\mu_4 = 162}$

Gateway Classes