Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Solution of Non Linear Partial Differential Equation by **CHARPIT METHOD**
- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise Live Doubt Classes
6	Unit wise Test
7	Unit wise set of PYQs with Answers
8	Course will be completed by 'Last week of Nov 2023'
	Tentative AKTU Semester Exam- Jan/Feb

Paid Courses are available in Gateway Classes Application
Link in Description

CHARPIT METHOD

It is a general method to solve non-linear first order partial differential equation

STEPS TO SOLVE

Step-I Write the given PDE in the form given below

$$f(x, y, z, p, q) = 0 \quad \text{---------} \ (1)$$

Step-II Partially differentiate equation (1) w.r.t. x, y, z, p and q to find,

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial f}{\partial z}, \quad \frac{\partial f}{\partial p}, \quad \frac{\partial f}{\partial q}$$

Step-III Write Charpit Auxiliary equation

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

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Step-IV Select two proper fraction so that resulting integral may be simplest relation involving at least one of p and q

Step-V With the help of relation in step 4 and equation (1) find p and q in terms of x, y, z

Step-VI Put the value of p and q in $\boxed{dz = pdx + qdy}$

Step-VII Integrate above equation to get required Answer.

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

p q

Q.1 By Charpit's method, find the complete solution of PDE : $px + qy = pq$

$$px + qy = pq$$

$$px + qy - pq = 0 \quad \text{—— (1)}$$

Let $f(x, y, z, p, q) = px + qy - pq = 0$

$$\frac{\partial f}{\partial x} = p, \quad \frac{\partial f}{\partial y} = q, \quad \frac{\partial f}{\partial z} = 0, \quad \frac{\partial f}{\partial p} = x - q, \quad \frac{\partial f}{\partial q} = y - p$$

Charpit Aux. Eqn

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

$$\frac{dx}{-(x-q)} = \frac{dy}{-(y-p)} = \frac{dz}{-p(x-q)-q(y-p)} = \frac{dp}{p+0} = \frac{dq}{q+0} \quad \text{— (2)}$$

$$\frac{dp}{p} = \frac{dq}{q}$$

Integrate both side

$$\int \frac{dp}{p} = \int \frac{dq}{q}$$

$$\log p = \log q + \log a$$

$$\log p = \log aq$$

$$p = aq \quad \text{— (3)}$$

Put p in (1)

$$px + qy - pq = 0$$

$$aqx + qy - aq^2 = 0$$

$$(ax+y)q = aq^2$$

$$aq = ax+y$$

$$q = \frac{ax+y}{a}$$

Put q in (3)

$$p = a \times \frac{(ax+y)}{a}$$

$$p = ax+y$$

We know that

$$dz = p\,dx + q\,dy$$

$$dz = (ax+y)\,dx + \left(\frac{ax+y}{a}\right)dy$$

$$\int dz = \int (ax+y)\,dx + \left(\frac{ax+y}{a}\right)dy$$

$$z = \int (ax+y)\left(dx + \frac{dy}{a}\right)$$

$$z = \int (ax+y)\left(\frac{adx+dy}{a}\right)$$

$$z = \frac{1}{a}\int (ax+y)(adx+dy)$$

Put $ax+y = t$

$$adx+dy = dt$$

$$z = \frac{1}{a}\int t\,dt = \frac{1}{a}\frac{t^2}{2} + b$$

$$\boxed{az = \frac{(ax+y)^2}{2} + b}$$

Q.2 Solve the following equation by Charpit's method : $pxy + pq + qy = yz$

$$pxy + pq + qy = yz$$

$$pxy + pq + qy - yz = 0 \quad \text{——①}$$

Let $f(x, y, z, p, q) = pxy + pq + qy - yz = 0$

$$\frac{\partial f}{\partial x} = py, \quad \frac{\partial f}{\partial y} = px + q - z, \quad \frac{\partial f}{\partial z} = -y, \quad \frac{\partial f}{\partial p} = xy + q, \quad \frac{\partial f}{\partial q} = p + y$$

Charpit Aux. Eqⁿ

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

$$\frac{dx}{-(ny+q)} = \frac{dy}{-(p+y)} = \frac{dz}{-p(ny+q)-q(p+y)} = \frac{dp}{py-py} = \frac{dq}{pn+q-z-qy}$$

$$\frac{dp}{0} = \frac{dq}{pn+q-z-qy}$$

$$dp = 0$$

$$\int dp = 0$$

$$\boxed{p = a} \quad -(3)$$

Put p in (1)

$$pny + pq + qy - yz = 0$$

$$any + aq + qy - yz = 0$$

$$(a+y)q = yz - any$$

$$\boxed{q = \frac{y(z-an)}{a+y}}$$

We know that

$$dz = p\,dx + q\,dy$$

$$dz = a\,dx + \frac{y(z-ax)}{a+y}\,dy$$

$$dz - a\,dx = \frac{y(z-ax)}{a+y}\,dy$$

$$\int \frac{dz - a\,dx}{z-ax} = \int \frac{y}{a+y}\,dy$$

$$\int \frac{dz - a\,dx}{z-ax} = \int \frac{a+y-a}{a+y}\,dy$$

$$\int \frac{dz - a\,dx}{z-ax} = \int \left(1 - \frac{a}{a+y}\right) dy$$

$$\boxed{\log(z-ax) = y - a\log(a+y) + b}$$

Q.3 Find the complete integral of PDE : $(p^2 + q^2)y = qz$

$$(p^2+q^2)y = qz$$

$$(p^2+q^2)y - qz = 0 \quad \text{— (1)}$$

Let $f(x, y, z, p, q) = (p^2+q^2)y - qz = 0$

$$\frac{\partial f}{\partial x} = 0 \,,\ \frac{\partial f}{\partial y} = p^2+q^2 \,,\ \frac{\partial f}{\partial z} = -q \,,\ \frac{\partial f}{\partial p} = 2py \,,\ \frac{\partial f}{\partial q} = 2qy - z$$

Charpit Aux. Eqn

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

$$\frac{dx}{-2py} = \frac{dy}{-(2qy-z)} = \frac{dz}{-2p^2y-q(2qy-z)} = \frac{dp}{-pq} = \frac{dq}{p^2+q^2-q^2}$$

$$\frac{dp}{-pq} = \frac{dq}{p^2}$$

$$\frac{dp}{-q} = \frac{dq}{p}$$

$$p\,dp = -q\,dq$$

$$p\,dp + q\,dq = 0$$

$$\int p\,dp + \int q\,dq = 0$$

$$\frac{p^2}{2} + \frac{q^2}{2} = \frac{a^2}{2}$$

$$p^2 + q^2 = a^2 \quad \text{— (3)}$$

Put eqn (3) in (1)

$$(p^2+q^2)y - qz = 0$$

$$a^2y - qz = 0$$

$$a^2y - qz = 0$$

$$\boxed{q = \frac{a^2y}{z}}$$

Put q in (3)

$$p^2 + \left(\frac{a^2y}{z}\right)^2 = a^2$$

$$p^2 = a^2 - \frac{a^4y^2}{z^2}$$

$$p^2 = \frac{a^2z^2 - a^4y^2}{z^2}$$

$$p^2 = \frac{a^2}{z^2}(z^2 - a^2y^2)$$

$$\boxed{p = \frac{a}{z}\sqrt{z^2 - a^2y^2}}$$

We know that

$$dz = p\,dx + q\,dy$$

$$dz = \frac{a}{z}\sqrt{z^2 - a^2y^2}\,dx + \frac{a^2y}{z}\,dy$$

$$z\,dz = a\left(\sqrt{z^2 - a^2y^2}\right)dx + a^2y\,dy$$

$$z\,dz - a^2y\,dy = a\left(\sqrt{z^2 - a^2y^2}\right)dx$$

$$\int \frac{z\,dz - a^2y\,dy}{\sqrt{z^2 - a^2y^2}} = \int a\,dx$$

$$z^2 - a^2y^2 = t$$

$$2z\,dz - 2a^2y\,dy = dt$$

$$z\,dz - a^2y\,dy = \frac{dt}{2}$$

$$\frac{1}{2}\int \frac{dt}{\sqrt{t}} = ax$$

$$\frac{1}{2}\int t^{-1/2}\,dt = ax$$

$$\frac{t^{1/2}}{\frac{1}{2}} = ax + b$$

$$t^{1/2} = ax + b$$

$$(z^2 - a^2y^2)^{1/2} = ax + b$$

$$\boxed{(z^2 - a^2y^2) = (ax + b)^2}$$

Q .1 ***Solve :*** $2zx - px^2 - 2pxy + pq = 0$

Ans $z = ay + b(x^2 - a)$

Q .2 ***Solve the following equation by charpit's method :*** $z^2 = pqxy$

Ans $z = ax^b y^{1/b}$

Q .3 ***Solve the following equation by charpit's method :*** $z = p^2x + q^2x$

Ans $\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b$

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Thank You

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