Engineering Mathematics

UNIT : Partial Differential Equation

Today's Target

- Non Linear Partial Differential Equation (Some Standard Forms)
- PYQs
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP with Answers
4	DPP Solutions
5	Unit wise Live Doubt Classes
6	Unit wise Test
7	Unit wise set of PYQs with Answers
8	Course will be completed by 'Last week of Nov 2023'
	Tentative AKTU Semester Exam- Jan/Feb

Paid Courses are available in Gateway Classes Application
Link in Description

Non – Linear Partial Differential Equations

Some Standard Forms

Type – I

Equation containing p and q only

$$f(p,\ q) = 0$$

Steps to solve

Step-I $f(p,\ q) = 0$ ——— (1)

Step-II Replace p by a and q by b

$$f(a,\ b) = 0 \quad \text{——— (2)}$$

Step-III Find b from (2) in terms of a

Step-IV Complete solution is given by

$$\boxed{z = ax + by + c} \quad \text{——— (3)}$$

Step-V Put b in (3)

Q.1 Solve : $p + q = 1$

$$p+q=1 \quad \text{---} \quad (1)$$

Replace p by a and q by b

$$a+b=1 \quad \text{---} \quad (2)$$

$$b=1-a$$

complete solution is

$$z=ax+by+c \quad \text{---} \quad (3)$$

$$\boxed{z=ax+(1-a)y+c}$$

Q.2 Solve : $\sqrt{p}+\sqrt{q}=1$

$$\sqrt{p}+\sqrt{q}=1 \quad \text{---} \quad (1)$$

Replace p by a and q by b

$$\sqrt{a}+\sqrt{b}=1 \quad \text{---} \quad (2)$$

$$\sqrt{b}=1-\sqrt{a}$$

$$b=(1-\sqrt{a})^2$$

complete solution is

$$z=ax+by+c$$

$$\boxed{z=ax+(1-\sqrt{a})^2 y+c}$$

Q.3 Solve the partial differential equation : $p^2 + p = q^2$

$$p^2 + p = q^2 \quad \text{—— (1)}$$

Replace p by a and q by b

$$a^2 + a = b^2 \quad \text{—— (2)}$$

$$b^2 = a^2 + a$$

$$b = \sqrt{a^2 + a}$$

Complete solution is

$$z = ax + by + c \quad \text{— (3)}$$

Put b in (3)

$$\boxed{z = ax + \left(\sqrt{a^2 + a}\right) y + c}$$

Type – II

Clairaut Form $z = px + qy + f(p, q)$

Steps to solve

Step-I $z = px + qy + f(p, q)$ ——— (1)

Step-II Replace p by a and q by b in (1)

Step-III Complete solution is given by

$$\boxed{z = ax + by + f(a, b)}$$

Q.4 Solve : $z = px + qy + \sqrt{1 + p^2 + q^2}$.

$$z = px + qy + \sqrt{1 + p^2 + q^2} \quad \text{---} (1)$$

Replace p by a and q by b

$$\boxed{z = ax + by + \sqrt{1 + a^2 + b^2}}$$

Q.5 Solve : $z = px + qy + \frac{p}{p+q}$.

$$z = px + qy + \frac{p}{p+q} \quad \text{---} (1)$$

Replace p by a and q by b

$$\boxed{z = ax + by + \frac{a}{a+b}}$$

Type – III

Equations not containing x and y

$$f(z,\ p,\ q) = 0$$

Steps to solve

Step-I Let $u = x + ay$ so that $p = \frac{dz}{du}$ $q = a\frac{dz}{du}$

Step-II Put the values of p and q in the given equation

Step-III Solve the ordinary differential equation in z and u

Step-IV Replace u by $x + ay$ in the solution of ODE

$$u = x + ay$$

$$\frac{\partial u}{\partial x} = 1 \qquad \frac{\partial u}{\partial y} = a$$

We know that

$$p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \times \frac{\partial u}{\partial x} = \frac{\partial z}{\partial u} = \frac{dz}{du}$$

$$q = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \times \frac{\partial u}{\partial y} = a\frac{\partial z}{\partial u} = a\frac{dz}{du}$$

Q.6 Solve the partial differential equation : $p + q = z$

$$p+q=z \quad \text{—①}$$

Let $u = x+ay$ so that

$$p = \frac{dz}{du} \quad \text{and} \quad q = a\frac{dz}{du}$$

Put p and q in ①

$$\frac{dz}{du} + a\frac{dz}{du} = z$$

$$(1+a)\frac{dz}{du} = z$$

$$(1+a)\frac{dz}{z} = du$$

Integrate both side

$$(1+a)\int\frac{dz}{z} = \int du$$

$$(1+a)\log z = u + b$$

Put $u = x+ay$

$$\boxed{(1+a)\log z = x+ay+b}$$

Q.7 Solve : $p(1 + q^2) = q\,(z-a)$

$$p(1+q^2) = q(z-a) \quad \text{—— ①}$$

Let $u = x + by$ so that

$$p = \frac{dz}{du} \quad \text{and} \quad q = b\frac{dz}{du}$$

Put p and q in ①

$$\frac{dz}{du}\left[1+\left(b\frac{dz}{du}\right)^2\right] = b\frac{dz}{du}(z-a)$$

$$1 + b^2\left(\frac{dz}{du}\right)^2 = bz - ab$$

$$b^2\left(\frac{dz}{du}\right)^2 = bz - ab - 1$$

$$b\frac{dz}{du} = \sqrt{bz-ab-1}$$

$$\frac{dz}{du} = \frac{1}{b}\sqrt{bz-ab-1}$$

$$\frac{b\,dz}{\sqrt{bz-ab-1}} = du$$

Integrate both side

$$\int \frac{b\,dz}{\sqrt{bz-ab-1}} = \int du$$

$$b\int (bz-ab-1)^{-\frac{1}{2}} = u + c$$

$$\frac{b(bz-ab-1)^{\frac{1}{2}}}{\frac{1}{2}\times b} = u + c$$

$$2(bz-ab-1)^{\frac{1}{2}} = u + c$$

$$4(bz-ab-1) = (u+c)^2$$

$$\boxed{4(bz-ab-1) = (x+by+c)^2}$$

Type – IV

Equations of the form

$$f_1(x,\ p) = f_2(y,\ q)$$

In these equations, z is absent and the terms containing x and p can be written on one side and terms containing y and q can be written on other side.

Steps to solve

Step-I $f_1(x,\ p) = f_2(y,\ q) = a$ ——— (1)

Step-II $f_1(x,\ p) = a$ *Solve it for* p ~~Let $p = f_1(x)$~~

Step-III $f_2(y,\ q) = a$ *Solve it for* q ~~Let $q = f_2(y)$~~

Step-IV We know that $\boxed{dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy = p\,dx + q\,dy}$

Step-V Put p and q in (2) and integrate it $z = \int F_1(x)dx + \int F_2(y)dy + C$

p q

Q.8 Solve : $p^2 - q^2 = x - y$

$$p^2 - q^2 = x - y$$

$$p^2 - x = q^2 - y$$

Let $p^2 - x = q^2 - y = a$ ——— (1)

$$p^2 - x = a$$

$$p^2 = x + a$$

$$p = \sqrt{x+a}$$

$$q^2 - y = a$$

$$q^2 = y + a$$

$$q = \sqrt{y+a}$$

We know that

$$dz = p\,dx + q\,dy$$

Put p and q in above equation

$$dz = \sqrt{x+a}\,dx + \sqrt{y+a}\,dy$$

Integrate both side

$$z = \int (x+a)^{1/2}\,dx + \int (y+a)^{1/2}\,dy$$

$$\boxed{z = \frac{2}{3}(x+a)^{3/2} + \frac{2}{3}(y+a)^{3/2} + c}$$

PDE : DPP-6

Topic : Non Linear Partial Differential Equation(Some Standard Forms)

Q .1 Solve : $p + q = pq$

Q .2 Solve the partial differential equation : $p^2 + q^2 = 2$

Q .3 Solve the partial differential equation $z = px + qy + \sin(p+q)$

Q .4 Solve : $9(p^2z + q^2) = 4$

Q .5 Solve : $p - 3x^2 = q^2 - y$

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Thank You

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